A5L

Tabelle:

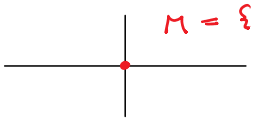
	0	30	45	60	90
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

A5: (Bereiche komplexer Zahlen) Zeichne die Mengen komplexer Zahlen (oder einen Ausschnitt davon):
a. $\{z \in \mathbb{C} : z = 8i\bar{z}\}$ b. $\{z \in \mathbb{C} : -e < 2z + 2\bar{z} < e\}$ c. $\{z \in \mathbb{C} : z^4 = 81i^2\}$

a. $M = \{z \in \mathbb{C} : z = 8i\bar{z}\}$ $\quad z = a+ib$

Setze $z = a+ib$. Dann: $z = 8i\bar{z} \Rightarrow a+ib = 8i(a-ib) = 8ia + 8b$

$\Rightarrow a = 8b$ und $b = 8a \Rightarrow a = 64a \Rightarrow a = 0, b = 0$

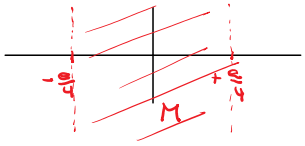


$M = \{0\}$

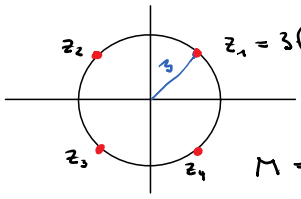
b. $M = \{z \in \mathbb{C} : -e < 2z + 2\bar{z} < e\}$ $\quad z = a+ib \Rightarrow 2z + 2\bar{z} = 2(a+ib+a-ib) = 4a$

$-e < 4a < e \Rightarrow -\frac{e}{4} < a < \frac{e}{4}$

$M = \{a+ib : -\frac{e}{4} < a < \frac{e}{4}\}$



c. $M = \{z \in \mathbb{C} : z^4 = 81i^2\}$ $\quad z^4 = 81i^2 = -81 \Rightarrow |z| = \sqrt[4]{81} = 3$



$z_1 = 3(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \frac{3}{2}\sqrt{2}(1+i)$

$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3}{4}\pi \Rightarrow z_2 = 3\cdot(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi) = \frac{3}{2}\sqrt{2}(-1+i)$

$\frac{\pi}{4} + \frac{4\pi}{4} = \frac{5}{4}\pi \Rightarrow z_3 = 3\cdot(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi) = \frac{3}{2}\sqrt{2}(-1-i)$

$\frac{\pi}{4} + \frac{6\pi}{4} = \frac{7}{4}\pi \Rightarrow z_4 = 3\cdot(\cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi) = \frac{3}{2}\sqrt{2}(1-i)$

$M = \{z_1, z_2, z_3, z_4\}$