A4L

Tabelle:

	0	30	45	60	90
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

A4: (Wurzeln komplexer Zahlen) Berechne jeweils alle $z \in \mathbb{C}$ mit:
a. $z^2 = (3-3i)^2$ b. $z^3 = \frac{64}{i}$ c. $z^4 = 16i^2$

a. $z^2 = (3-3i)^2 = 9 - 18i - 9 = -18i$

$|z^2| = 18 \Rightarrow |z| = \sqrt{18} = 3\cdot\sqrt{2}$

$z_1 = 3\cdot\sqrt{2}\cdot\left(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right) = 3\cdot\sqrt{2}\left(-\frac{1}{2}\sqrt{2} + i\,\frac{1}{2}\sqrt{2}\right) = -3+3i$

$z_2 = 3\cdot\sqrt{2}\cdot\left(\cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi\right) = 3\cdot\sqrt{2}\left(\frac{1}{2}\sqrt{2} - i\,\frac{1}{2}\sqrt{2}\right) = 3-3i$

b. $z^3 = \frac{64}{i} = \frac{64i}{i^2} = -64i$ $\quad |z^3| = 64 \Rightarrow |z| = 4$ $\quad \arg(-i) \hat{=} \frac{3}{2}\pi$

$z_1 = 4\cdot\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$\frac{\pi}{2} + \frac{2\pi}{3} = \frac{7}{6}\pi \Rightarrow z_2 = 4\cdot\cos\left(\frac{7}{6}\pi + i\sin\frac{7}{6}\pi\right)$

$\frac{\pi}{2} + \frac{4\pi}{3} = \frac{11}{6}\pi \Rightarrow z_3 = 4\cdot\cos\left(\frac{11}{6}\pi + i\sin\frac{11}{6}\pi\right)$

c. $z^4 = 16i^2 = -16$ $\quad |z^4| = 16 \Rightarrow |z| = 2$

$z_1 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3}{4}\pi \Rightarrow z_2 = 2\cdot\left(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right)$

$\frac{\pi}{4} + \frac{4\pi}{4} = \frac{5}{4}\pi \Rightarrow z_3 = 2\cdot\left(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi\right)$

$\frac{\pi}{4} + \frac{6\pi}{4} = \frac{7}{4}\pi \Rightarrow z_4 = 2\cdot\left(\cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi\right)$