A3L

A3: (Polardarstellung komplexer Zahlen)
Gib die folgenden komplexen Zahlen in Polardarstellung an und berechne jeweils Real und Imaginärteil.
a. $1+i$ b. $8\cos(\frac{\pi}{6}) + 8i\sin(\frac{\pi}{6})$ c. $-\sqrt{3}+3i$
d. $(1+2i)\cdot(3-i)$ e. $i\cdot\overline{3-4i}$ f. $(1+i)^{20}$

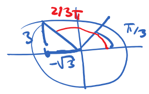
Tabelle:

	0	30	45	60	90
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

a. $z = 1+i \Rightarrow Re(z) = 1,\ Im(z) = 1$
polar: $|z| = \sqrt{2}$ $\arg(z) = \frac{\pi}{4} \Rightarrow z = \sqrt{2}\,(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

b. $z = 8\cdot\cos\frac{\pi}{6} + 8i\sin\frac{\pi}{6}$ polar: $z = 8\cdot(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$
$Re(z) = 8\cdot\frac{1}{2}\sqrt{3} = 4\cdot\sqrt{3}$, $Im(z) = \frac{8}{2} = 4$

c. $z = -\sqrt{3}+3i \Rightarrow Re(z) = -\sqrt{3},\ Im(z) = 3$
polar: $|z| = \sqrt{3+9} = \sqrt{12} = 2\cdot\sqrt{3}$ $\cos\alpha = \frac{-\sqrt{3}}{2\cdot\sqrt{3}} = -\frac{1}{2} \Rightarrow \alpha = \frac{2}{3}\pi$
$z = 2\cdot\sqrt{3}\,(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi)$



d. $z = (1+2i)(3-i) = 3-i+6i+2 = 5+5i \Rightarrow Re(z) = Im(z) = 5$
polar: $|z| = \sqrt{50}$, $z = \sqrt{50}\cdot(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

e. $z = i\cdot\overline{3-4i} = i(3+4i) = 3i-4 \Rightarrow Re(z) = -4,\ Im(z) = 3$
polar: $|z| = \sqrt{16+9} = 5$ $\cos\alpha = \frac{-4}{5} \Rightarrow \alpha = \cos^{-1}(-\frac{4}{5}) \approx 2.498$
$z = 5\cdot(\cos\alpha + i\sin\alpha)$

f. $z = (1+i)^{20} \Rightarrow |z| = (\sqrt{2})^{20} = 2^{10} = 1024$, $\alpha = 20\cdot\frac{\pi}{4} = 5\pi \Rightarrow \arg(z) = \pi$
polar: $z = 1024(\cos\pi + i\sin\pi)$, $Re(z) = -1024$, $Im(z) = 0$