A1L

A1: (Kehrwerte und Quotienten komplexer Zahlen bilden)
Berechne $\frac{1}{z}$ bzw. $\frac{w}{z}$ für:
a. $z = 4i$ b. $3 - i$ c. $z = 6\cos(\frac{\pi}{6}) + 6i\sin(\frac{\pi}{6})$
d. $z = -8i$, $w = 3 + 16i$ e. $z = i + 3, w = 6i$ f. $z = 2\cos(\pi) + 2i\sin(\pi)$, $w = 8 - 5i$

Tabelle:

	0	30	45	60	90
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

a. $z = 4i \Rightarrow \frac{1}{z} = \frac{1}{4i} \cdot \frac{i}{i} = -\frac{1}{4}i$

b. $z = 3 - i \Rightarrow \frac{1}{z} = \frac{1}{3-i} \cdot \frac{(3+i)}{3+i} = \frac{3+i}{10} = \frac{3}{10} + i\frac{1}{10}$

c. $z = 6 \cdot \cos\frac{\pi}{6} + 6i \sin\frac{\pi}{6} = 6 \cdot \frac{1}{2}\sqrt{3} + 6i\frac{1}{2} = 3\sqrt{3} + 3i = 3(\sqrt{3} + i)$

$\Rightarrow \frac{1}{z} = \frac{\sqrt{3} - i}{3(\sqrt{3}+i)\cdot(\sqrt{3}-i)} = \frac{\sqrt{3}-i}{3 \cdot 4} = \frac{\sqrt{3}}{12} - i\frac{1}{12}$

oder geometrisch: $z = 6 \cdot (\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) \Rightarrow |\frac{1}{z}| = \frac{1}{6}$ und $\arg(z) = -\frac{\pi}{6}$

$\Rightarrow \frac{1}{z} = \frac{1}{6}(\cos(-\frac{\pi}{6}) + i \cdot \sin(-\frac{\pi}{6})) = \frac{1}{6}(\frac{1}{2}\sqrt{3} - i\frac{1}{2}) = \frac{\sqrt{3}}{12} - i\frac{1}{12}$

d. $z = -8i$, $w = 3 + 16i \Rightarrow \frac{w}{z} = \frac{3+16i}{-8i} \cdot \frac{i}{i} = \frac{3i - 16}{8} = -2 + \frac{3}{8}i$

e. $z = i + 3$, $w = 6i \Rightarrow \frac{w}{z} = \frac{6i}{i+3} \cdot \frac{3-i}{3-i} = \frac{18i+6}{10} = \frac{3}{5} + \frac{9}{5}i$

f. $z = \underbrace{2\cos\pi + 2i\sin\pi}_{\text{geometrisch} \Rightarrow z = -2}$, $w = 8 - 5i \Rightarrow \frac{w}{z} = \frac{8-5i}{-2} = -4 + \frac{5}{2}i$