Lösungen A1-A6

Tabelle für sin/cos

0	30	45	60	90
0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

$\frac{1}{2}\sqrt{k}$

A1 a)

$$\int_0^{\pi/4} \underbrace{2x}_{u}\cdot\underbrace{\sin x}_{v'}\,dx = \Big[\underset{u}{2x}\cdot\underset{v}{(-\cos x)}\Big]_0^{\frac{\pi}{4}} - \int_0^{\pi/4} \underset{u'}{2}\cdot\underset{v}{(-\cos x)}\,dx$$

$$= -\frac{\pi}{2}\cdot\frac{1}{2}\sqrt{2} + 0 + \Big[2\cdot\sin x\Big]_0^{\pi/4}$$

$$= -\frac{\pi}{4}\sqrt{2} + \left(2\cdot\frac{1}{2}\sqrt{2} - 0\right)$$

$$= \underline{-\frac{\pi}{4}\sqrt{2} + \sqrt{2}}$$

b)

$$\int_0^3 \underbrace{\frac{2}{3}x}_{u}\cdot\underbrace{e^{2x}}_{v'}\,dx = \Big[\underset{u}{\frac{2}{3}x}\cdot\underset{v}{\frac{1}{2}e^{2x}}\Big]_0^3 - \int_0^3 \underset{u'}{\frac{2}{3}}\cdot\underset{v}{\frac{1}{2}e^{2x}}\,dx$$

$$= e^6 - 0 - \Big[\frac{1}{3}\cdot\frac{1}{2}e^{2x}\Big]_0^3$$

$$= e^6 - \frac{1}{6}e^6 + \frac{1}{6} = \underline{\frac{5}{6}e^6 + \frac{1}{6}}$$

c)

$$\int_0^4 \underbrace{x}_{u}\cdot\underbrace{(x-2)^5}_{v'}\,dx = \Big[\underset{u}{x}\cdot\underset{v}{\frac{1}{6}(x-2)^6}\Big]_0^4 - \int_0^4 \underset{u'}{1}\cdot\underset{v}{\frac{1}{6}(x-2)^6}\,dx$$

$$= \frac{4}{6}2^6 - 0 - \Big[\frac{1}{6}\cdot\frac{1}{7}(x-2)^7\Big]_0^4$$

$$= \frac{2^7}{3} - \frac{1}{42}2^7 + \frac{1}{42}(-2)^7$$

$$= 2^7\left(\frac{1}{3} - \frac{1}{42} - \frac{1}{42}\right) = 2^7\cdot\frac{12}{42} = 2^7\cdot\frac{2}{7} = \underline{\frac{2^8}{7}}$$

A2 a)

$$\int_0^2 \underbrace{3x^2}_{u}\,\underbrace{e^x}_{v'}\,dx = \Big[\underset{u}{3x^2}\cdot\underset{v}{e^x}\Big]_0^2 - \int_0^2 \underbrace{6x}_{u' \,/\, u}\,\underbrace{e^x}_{v \,/\, v'}\,dx$$

$$= 12e^2 - 0 - \left(\Big[\underset{u}{6x}\cdot\underset{v}{e^x}\Big]_0^2 - \int_0^2 \underset{u'}{6}\underset{v}{e^x}\,dx\right)$$

$$= 12e^2 - (12e^2 - 0) + \Big[6e^x\Big]_0^2$$

$$= \underline{6e^2 - 6}$$

$$= \underline{6e^{6} - 6}$$

b) $$\int_0^{\pi} \underbrace{x^2}_{u} \underbrace{\cos(\tfrac{1}{2}x)}_{v'}\, dx = \Big[\underbrace{x^2}_{u} \cdot \underbrace{2\sin(\tfrac{1}{2}x)}_{v}\Big]_0^{\pi} - \int_0^{\pi} \underbrace{\overset{u'}{2x}}_{u} \cdot \underbrace{\overset{v}{2\sin(\tfrac{1}{2}x)}}_{v'}\, dx$$

$$= 2\pi^2 - 0 - \left(\Big[\underbrace{4x}_{u} \cdot \underbrace{(-2\cos(\tfrac{1}{2}x))}_{v}\Big]_0^{\pi} - \int_0^{\pi} \underbrace{4}_{u'} \cdot \underbrace{(-2\cos(\tfrac{1}{2}x))}_{v}\, dx \right)$$

$$= 2\pi^2 - \left((-8\pi \cdot 0 + 0) - \Big[-16 \sin(\tfrac{1}{2}x)\Big]_0^{\pi} \right)$$

$$= 2\pi^2 - \big(-(-16+0)\big) = \underline{2\pi^2 - 16}$$

c) $$\int_0^{5/2} \underbrace{x^2}_{u} \cdot \underbrace{(2x-5)^4}_{v'}\, dx = \Big[\underbrace{x^2}_{u} \cdot \underbrace{\tfrac{1}{5}(2x-5)^5}_{v}\Big]_0^{5/2} - \int_0^{5/2} \underbrace{\overset{u'}{2x}}_{u} \cdot \underbrace{\overset{v}{\tfrac{1}{5}(2x-5)^5}}_{v'}\, dx$$

$$= 0 - 0 - \left(\Big[\underbrace{\tfrac{2}{5}x}_{u}\ \underbrace{\tfrac{1}{6}(2x-5)^6 \cdot \tfrac{1}{2}}_{v}\Big]_0^{5/2} - \int_0^{5/2} \underbrace{\tfrac{2}{5}}_{u'} \cdot \underbrace{\tfrac{1}{6}(2x-5)^6 \cdot \tfrac{1}{2}}_{v}\, dx \right)$$

$$= -\left(0 - 0 - \Big[\tfrac{1}{60} \cdot \tfrac{1}{7}(2x-5)^7 \cdot \tfrac{1}{2}\Big]_0^{5/2} \right)$$

$$= -\frac{1}{120} \cdot \frac{1}{7} \cdot (-5)^7 = \frac{1}{24} \cdot \frac{1}{7} \cdot 5^6 = \underline{\frac{5^6}{168}}$$

A3

a) $$\int \underbrace{2x}_{v'} \underbrace{\ln(x)}_{u} = \underbrace{x^2}_{v} \cdot \underbrace{\ln(x)}_{u} - \int \underbrace{x^2}_{v} \cdot \underbrace{\tfrac{1}{x}}_{u'}\, dx + C$$

$$= x^2 \cdot \ln(x) - \int x\, dx + C$$

$$= \underline{x^2 \cdot \ln(x) - \tfrac{1}{2}x^2} + C$$

Hinweis:
Für die Gesamtheit aller Stammfunktionen verwendet man häufig die Schreibweise

$$\int x^2\, dx = \tfrac{1}{3}x^3 + C$$

b) $$\int \underbrace{x^2}_{v'} \cdot \underbrace{\ln(x)}_{u}\, dx = \underbrace{\tfrac{1}{3}x^3}_{v} \cdot \underbrace{\ln(x)}_{u} - \int \underbrace{\tfrac{1}{3}x^3}_{v} \cdot \underbrace{\tfrac{1}{x}}_{u'}\, dx + C$$

$$= \tfrac{1}{3}x^3 \cdot \ln(x) - \int \tfrac{1}{3}x^2\, dx + C$$

$$= \underline{\tfrac{1}{3}x^3 \cdot \ln(x) - \tfrac{1}{9}x^3 + C}$$

c) $$\int \underbrace{\tfrac{1}{x^2}}_{v'} \underbrace{\ln(x)}_{u}\, dx = \underbrace{-\tfrac{1}{x}}_{v} \cdot \underbrace{\ln(x)}_{u} - \int -\underbrace{\tfrac{1}{x}}_{v} \cdot \underbrace{\tfrac{1}{x}}_{u'} \cdot dx + C$$

$$= -\tfrac{1}{x} \ln(x) + \int \tfrac{1}{x^2}\, dx + C$$

$$= -\tfrac{1}{x} \ln(x) - \tfrac{1}{x} + C$$

A4

a. $$\underbrace{\int_0^\pi \cos^2 x\,dx}_{=A} = \int_0^\pi \underset{u}{\cos x}\cdot\underset{v'}{\cos x}\,dx = \Big[\underset{u}{\cos x}\cdot\underset{v}{\sin x}\Big]_0^\pi - \int_0^\pi \underset{u'}{(-\sin x)}\,\underset{v}{\sin x}\,dx$$

$$= 0 - 0 + \int_0^\pi \sin^2 x\,dx$$

$(\sin^2 x + \cos^2 x = 1)$

$$= \int_0^\pi (1-\cos^2 x)\,dx$$

$$2A = \int_0^\pi 1\,dx = \pi \Rightarrow \underline{A = \frac{\pi}{2}}$$

b. $$\int_{-1}^1 \sin^2(\pi x)\,dx = \int_{-1}^1 \underset{u}{\sin(\pi x)}\cdot\underset{v'}{\sin(\pi x)}\,dx = \Big[\underset{u}{\sin(\pi x)}\cdot\underset{v}{\tfrac{1}{\pi}(-\cos(\pi x))}\Big]_{-1}^1 - \int_{-1}^1 \underset{u'}{\pi\cdot\cos(\pi x)}\cdot\underset{v}{\tfrac{1}{\pi}(-\cos(\pi x))}\,dx$$

$$= 0 - 0 + \int_{-1}^1 \cos^2(\pi x) = \int_{-1}^1 (1-\sin^2(\pi x))\,dx$$

$$= \frac{1}{2}\int_{-1}^1 1\,dx = \frac{1}{2}\cdot 2 = \underline{1}$$

c. $$\int_1^2 \underset{v'}{\frac{1}{x}}\,\underset{u}{\ln x}\,dx = \Big[\underset{v}{\ln(x)}\cdot\underset{u}{\ln(x)}\Big]_1^2 - \int_1^2 \underset{v}{\ln x}\cdot\underset{u'}{\frac{1}{x}}\,dx$$

$$= \frac{1}{2}\Big[\ln^2(x)\Big]_1^2 = \frac{1}{2}\left(\ln^2(2) - \ln^2(1)\right) = \underline{\frac{1}{2}\ln^2(2)}$$

A5

a) $$\int_0^1 3e^{2x-1}\,dx = \Big[3\cdot\frac{1}{2}e^{2x-1}\Big]_0^1 = \frac{3}{2}e - \frac{3}{2}e^{-1} = \underline{\frac{3e}{2} - \frac{3}{2e}}$$

b) $$\int_1^2 (3-2x)^2\,dx = \Big[-\frac{1}{2}\cdot\frac{1}{3}(3-2x)^3\Big]_1^2 = -\frac{1}{6}(-1)^3 + \frac{1}{6}1^3 = \underline{\frac{1}{3}}$$

c) $$\int_0^2 \sqrt{4x+1}\,dx = \Big[\frac{1}{4}\cdot\frac{2}{3}(4x+1)^{\frac{3}{2}}\Big]_0^2 = \frac{1}{6}\cdot 9^{\frac{3}{2}} - \frac{1}{6}\cdot 1^{\frac{3}{2}} = \frac{27}{6} - \frac{1}{6} = \underline{\frac{13}{3}}$$

A6

a) $$\int_0^1 \frac{2e^x}{2e^x+1}\,dx = \Big[\ln|2e^x+1|\Big]_0^1 = \underline{\ln(2e+1) - \ln(3)}$$

Die Nenner in A6 haben im Integrationsbereich keine Nullstellen.

b) $$\int_1^2 \frac{2x+2}{x^2+2x+3}\,dx = \Big[\ln|x^2+2x+3|\Big]_1^2 = \ln 11 - \ln 6$$

c) $$\int_1^2 \frac{x^2}{1-8x^3}\,dx = \Big[-\frac{1}{24}\ln|1-8x^3|\Big]_1^2 = -\frac{1}{24}\ln|-63| + \frac{1}{24}\ln|-7|$$

$$= \frac{1}{24}(\ln 7 - \ln 63) = \frac{1}{24}\ln\left(\frac{7}{63}\right)$$