1) $\underbrace{\sqrt{3-0.5x}}_{\geq 0} < \underbrace{x-3}_{\geq 0 \text{ für } x \geq 3}$

Definitionsmenge:

$3-0.5x \geq 0$

$3 \geq 0.5x$

$6 \geq x$

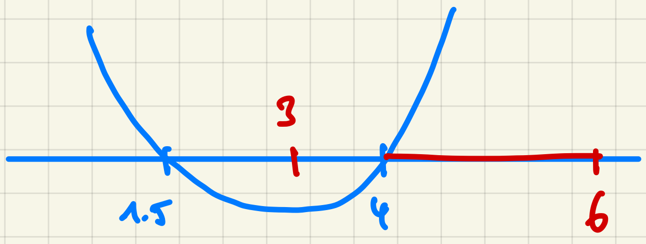
$D = (-\infty, 6]$

1. Fall: $x \geq 3$

$(3-0.5x) < x^2-6x+9$

$0 < x^2-5.5x+6$

Vieta: $x_1 = 1.5$, $x_2 = 4$



$L_1 = (4;6]$

2. Fall: $x < 3$ $\quad L_2 = \{\}$

$\underline{L = (4;6]}$

b) $\underbrace{\sqrt{x^2+5}}_{\geq 0} \geq \underbrace{2x-1}_{\geq 0 \text{ für } x \geq \frac{1}{2}}$

Definitionsmenge: $D = \mathbb{R}$

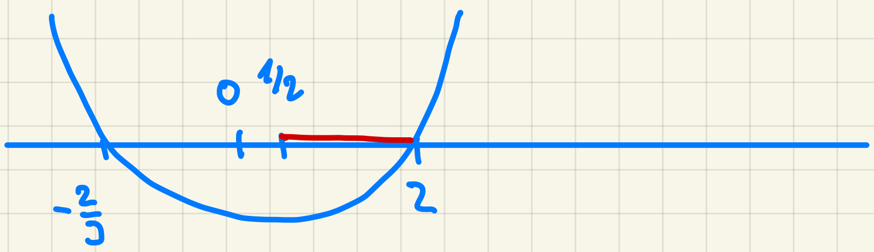
1. Fall $x \geq \frac{1}{2}$:

$x^2+5 \geq 4x^2-4x+1$

$0 \geq 3x^2-4x-4$

$x_{1/2} = \dfrac{4 \pm \sqrt{16+48}}{6} = \dfrac{4 \pm 8}{6}$

$x_1 = 2$, $x_2 = -\frac{2}{3}$



$L_1 = [\frac{1}{2}; 2]$

2. Fall $x < \frac{1}{2}$: $\sqrt{x^2+5} \geq \underbrace{2x-1}_{<0}$

wahre Aussage!

$L_2 = (-\infty, \frac{1}{2})$

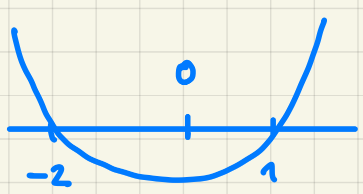
$\underline{L = (-\infty, 2]}$

c) $\underbrace{\sqrt{x} \cdot \sqrt{x+1}}_{\geq 0} \leq \underbrace{\sqrt{2}}_{\geq 0}$

$D = [0; \infty)$

$x(x+1) \leq 2$

$x^2+x-2 \leq 0$



Vieta: $x_1 = 1$, $x_2 = -2$

$L = [0; 1]$

d) $\underbrace{\sqrt{2x+6}}_{\geq 0} \leq \underbrace{0.25x+3}_{\geq 0 \text{ für } x \geq -12}$ Definitionsbereich: (*) $2x+6 \geq 0$, $2x \geq -6$, $x \geq -3$ $D = [-3;\infty)$

$2x+6 \leq \frac{1}{16}x^2 + \frac{3}{2}x + 9$ r.S: $\frac{1}{4}x + 3 \geq 0$, $1/4\, x \geq -3$, $x \geq -12$

$\frac{1}{16}x^2 - \frac{1}{2}x + 3 \geq 0$

$x_{1/2} = \dfrac{1/2 \pm \sqrt{1/4 - 3/4}}{1/8}$ keine Lösung.

Parabel bleibt immer oberhalb x-Achse

$L = [-3;\infty)$

(*): Äquivalenzumformung für alle $x \in D$. Daher keine Fallunterscheidung nötig.

2 a) $\sqrt{5 - \frac{5}{2}x} - \sqrt{2 - \frac{1}{2}x} \leq 1$ Definitionsmenge: $5 - \frac{5}{2}x \geq 0$, $5 \geq 5/2\, x$, $2 \geq x$; $2 - 1/2\, x \geq 0$, $2 \geq 1/2\, x$, $4 \geq x$

$\underbrace{\sqrt{5 - \frac{5}{2}x}}_{\geq 0} \leq \underbrace{1 + \sqrt{2 - 1/2\, x}}_{\geq 0}$ $|()^2$ $D = (-\infty, 2]$

$5 - \frac{5}{2}x \leq 1 + 2\sqrt{2 - 1/2\, x} + 2 - 1/2\, x$

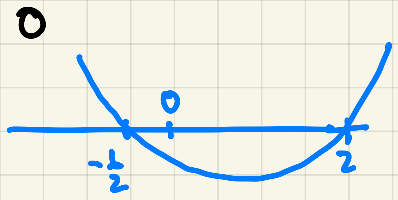
$2 - 2x \leq 2\sqrt{2 - 1/2\, x}$ $|:2$

$\underbrace{1 - x}_{\geq 0 \text{ für } x \leq 1} \leq \underbrace{\sqrt{2 - 1/2\, x}}_{\geq 0}$

1. Fall: $x \leq 1$: $1 - 2x + x^2 \leq 2 - 1/2\, x$

$x^2 - \frac{3}{2}x - 1 \leq 0$

Vieta: $x_1 = -\frac{1}{2}$, $x_2 = 2$



$L_1 = [-\frac{1}{2}; 2]$

2. Fall: $x > 1$:

$\underbrace{1 - x}_{< 0} \leq \underbrace{\sqrt{2 - 0.5x}}_{\geq 0}$ wahre Aussage $\Rightarrow L_2 = (1, 2]$ $L = [-\frac{1}{2}; 2]$

b) $2\sqrt{8 - 2x} > 5 + \sqrt{\frac{1}{2}x - 3}$ Definitionsmenge: $8 - 2x \geq 0$, $-2x \geq -8$, $x \leq 4$; $1/2\, x - 3 \geq 0$, $1/2\, x \geq 3$, $x \geq 6$

$D = \{\ \} \Rightarrow L = \{\ \}$

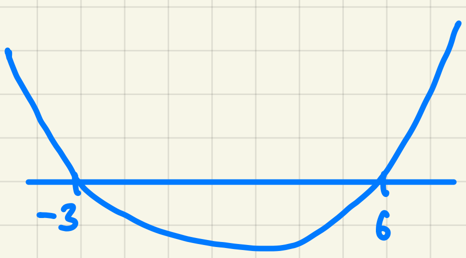
c) $\sqrt{5x + 19} - 3 < \sqrt{22 - x}$ Definitionsmenge: $5x + 19 \geq 0$, $5x \geq -19$, $x \geq -19/5$; $22 - x \geq 0$, $22 \geq x$ $D = [-\frac{19}{5}; 22]$

$\underbrace{\sqrt{5x + 19}}_{\geq 0} < \underbrace{\sqrt{22 - x} + 3}_{\geq 0}$

$5x + 19 < 22 - x + 6\sqrt{22 - x} + 9$

$6x - 12 < 6\sqrt{22 - x}$ $|:6$

$\underbrace{x - 2}_{\geq 0 \text{ für } x \geq 2} < \underbrace{\sqrt{22 - x}}_{\geq 0}$



1. Fall: $x \geq 2$

$x^2 - 4x + 4 < 22 - x$

$x^2 - 3x - 18 < 0$ Vieta: $x_1 = 6$, $x_2 = -3$ $L_1 = [2, 6)$

2. Fall: $x < 2$

$x - 2 < \sqrt{22 - x}$ wahre Aussage: $L_2 = [-\frac{19}{5}; 2)$ $L = [-\frac{19}{5}; 6)$

② d) $\underbrace{\sqrt{8-2x}}_{\geq 0} \geq \underbrace{\sqrt{17-4x}+1}_{\geq 0}$

Definitionsmenge: $8-2x \geq 0$ $\quad$ $17-4x \geq 0$

$8 \geq 2x$ $\quad$ $-4x \geq -17$

$4 \geq x$ $\quad$ $x \leq 17/4$

$\mathbb{D} = (-\infty, 4]$

$8-2x \geq 17-4x+2\sqrt{17-4x}+1$

$2x-10 \geq 2\sqrt{17-4x} \quad |:2$

$\underbrace{x-5}_{\geq 0 \text{ für } x \geq 5} \geq \underbrace{\sqrt{17-4x}}_{\geq 0}$

1. Fall: $x \geq 5$. $x \notin \mathbb{D} \Rightarrow L_1 = \{\}$

2. Fall: $x < 5$: falsche Aussage $\Rightarrow L_2 = \{\}$ $\quad$ $\underline{L = \{\}}$

e) $\sqrt{12x+1}-4 \geq \sqrt{4x-7}$

Definitionsmenge: $12x+1 \geq 0$ $\quad$ $4x-7 \geq 0$

$12x \geq -1$ $\quad$ $4x \geq 7$

$x \geq -1/12$ $\quad$ $x \geq 7/4$

$\underbrace{\sqrt{12x+1}}_{\geq 0} \geq \underbrace{\sqrt{4x-7}+4}_{\geq 0} \quad |()^2$ $\quad$ $\mathbb{D} = [7/4; \infty)$

$12x+1 \geq 4x-7+8\sqrt{4x-7}+16$

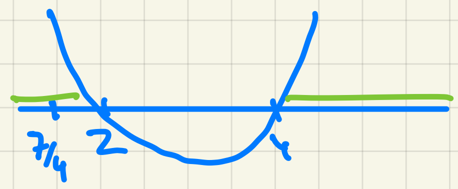
$8x-8 \geq 8\sqrt{4x-7} \quad |:8$

$\underbrace{x-1}_{\geq 0 \text{ für } x \geq 1} \geq \underbrace{\sqrt{4x-7}}_{\geq 0}$

1. Fall $x \geq 1$:

$x^2-2x+1 \geq 4x-7$

$x^2-6x+8 \geq 0$ $\quad$ Vieta: $x_1 = 2$, $x_2 = 4$



$L_1 = [7/4; 2] \cup [4, \infty)$

2. Fall $x < 1 \Rightarrow x \notin \mathbb{D} \Rightarrow L_2 = \{\}$ $\quad$ $\underline{L = [7/4; 2] \cup [4, \infty)}$

③ $\sqrt{x^2+2x-4}-1-\sqrt{x^2-7} > 0$

Definitionsmenge: $x^2+2x-4 \geq 0$

$x_{1/2} = \frac{-2 \pm \sqrt{4+16}}{2} = \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2}$

$x_1 = -1-\sqrt{5}$, $x_2 = -1+\sqrt{5}$

$\mathbb{D}_1 = (-\infty, -1-\sqrt{5}] \cup [-1+\sqrt{5}; \infty)$

$x^2-7 > 0$

$x^2 > 7$ $\quad$ $x_{1/2} = \pm\sqrt{7}$

$\mathbb{D}_2 = (-\infty, -\sqrt{7}] \cup [\sqrt{7}, \infty)$

$\mathbb{D} = (-\infty, -1-\sqrt{5}] \cup [\sqrt{7}, \infty)$

$\underbrace{\sqrt{x^2+2x-4}}_{\geq 0} > \underbrace{1+\sqrt{x^2-7}}_{\geq 0} \quad |()^2$

$\cancel{x^2}+2x-4 > 1+2\sqrt{x^2-7}+\cancel{x^2}-7$

$2x+2 > 2\sqrt{x^2-7} \quad |:2$

$\underbrace{x+1}_{\geq 0 \text{ für } x \geq -1} > \underbrace{\sqrt{x^2-7}}_{\geq 0}$

1. Fall $x \geq -1$:

$x^2+2x+1 > x^2-7$

$2x > -8$

$x > -4$ $\quad$ $L_1 = (-4; -1-\sqrt{5}] \cup [\sqrt{7}; \infty)$

2. Fall $x < 1$: $\underbrace{x+1}_{\leq 0} > \underbrace{\sqrt{x^2+7}}_{\geq 0}$ $\quad$ $L_2 = \{\}$

$\underline{L = (-4; -1-\sqrt{5}] \cup [\sqrt{7}; \infty)}$