Erarbeitung - Aufgabe

a) $x - \sqrt{25-6x} = 3$

Definitionsmenge: $25-6x \geq 0$

$25 \geq 6x$

$\frac{25}{6} \geq x$ $\qquad D = (-\infty; \frac{25}{6}]$

$$\begin{aligned} x-3 &= \sqrt{25-6x} \quad | ()^2 \\ x^2-6x+9 &= 25-6x \\ x^2 &= 16 \qquad x_{1/2} = \pm 4 \in D. \end{aligned}$$

Probe: $x_1 = 4$: $4 - \sqrt{1} = 3$ ✓

$x_2 = -4$: $-4 - \sqrt{49} = 3$ ↯ $\qquad$ Also: $\underline{L = \{4\}}$

b) $\sqrt{2x+1} - \sqrt{x-3} = 2$

Definitionsmenge: $2x+1 \geq 0$ $\qquad x-3 \geq 0$ $\qquad D = [3; +\infty)$

$2x \geq -1$ $\qquad x \geq 3$

$x \geq -\frac{1}{2}$

$$\begin{aligned} \sqrt{2x+1} - \sqrt{x-3} &= 2 \\ \sqrt{2x+1} &= 2 + \sqrt{x-3} \quad | ()^2 \\ 2x+1 &= 4 + 4\sqrt{x-3} + x-3 \\ x &= 4\sqrt{x-3} \quad | ()^2 \\ x^2 &= 16(x-3) = 16x - 48 \\ x^2 - 16x + 48 &= 0 \end{aligned}$$

Vieta: $x_1 = 4$, $x_2 = 12$. $\quad x_1, x_2 \in D$.

Probe $x_1 = 4$: $\sqrt{2 \cdot 4+1} - \sqrt{4-3} = 2$

$3 - 1 = 2$ ✓

$x_2 = 12$: $\sqrt{2 \cdot 12+1} - \sqrt{12-3} = 2$

$5 - 3 = 2$ ✓ $\qquad \underline{L = \{4; 12\}}$

① a) $x+3 = \sqrt{11x+9}$

Definitionsmenge: $11x+9 > 0$

$x > -\frac{9}{11}$

$x^2+6x+9 = 11x+9$

$x^2-5x = 0$

$x(x-5) = 0 \qquad x_1=0 \in D,\ x_2=5 \in D$

Probe: $x_1 = 0:\quad 3 = \sqrt{9}$ ✓

$x_2 = 5:\quad 8 = \sqrt{64}$ ✓ $\qquad \underline{L=\{0;5\}}$

b) $\sqrt{3x^2-20}-6=-2$ $\qquad$ Definitionsmenge: $3x^2-20 \geq 0$

$x^2 \geq \frac{20}{3}$

$D = (-\infty; -\sqrt{\frac{20}{3}}] \cup [\sqrt{\frac{20}{3}}; +\infty)$

$\sqrt{3x^2-20} = 4 \quad |()^2$

$3x^2-20 = 16$

$3x^2 = 36$

$x^2 = 12$

$x_{1/2} = \pm\sqrt{12} \in D$

Probe: $x_1 = \sqrt{12}:\quad \sqrt{3\cdot 12-20} = \sqrt{16} = 4$ ✓ $\qquad \underline{L = \{\pm\sqrt{12}\}}$

$x_2 = -\sqrt{12}:$ 〃

c) $\frac{x-11}{\sqrt{14}} = \sqrt{11-x}$ $\qquad$ Definitionsbereich: $11-x \geq 0$

$11 \geq x$

$D = (-\infty; 11]$

$\frac{x^2-22x+121}{14} = 11-x$

$x^2-22x+121 = 154-14x$

$x^2-8x-33 = 0$

Vieta: $x_1 = 11,\ x_2 = -3 \qquad x_1, x_2 \in D.$

Probe: $x_1 = 11:\quad 0 = 0$ ✓

$x_2 = -3:\quad \frac{-14}{\sqrt{14}} = \sqrt{14}$ ↯ $\qquad \underline{L = \{11\}}$

d) $3\sqrt{4+2x} = \sqrt{6-3x}$ $\qquad$ Definitionsbereich: $4+2x \geq 0$, $2x \geq -4$, $x \geq -2$ $\qquad$ $6-3x \geq 0$, $-3x \geq -6$, $x \leq 2$

$D = [-2; 2]$

$9(4+2x) = 6-3x$

$36+18x = 6-3x$

$21x = -30$

$x = -\frac{30}{21} = -\frac{10}{7} \in D$

Probe: linke Seite: $3\sqrt{4-\frac{20}{7}} = 3\sqrt{\frac{8}{7}}$

rechte Seite: $\sqrt{6+\frac{30}{7}} = \sqrt{\frac{72}{7}} = \sqrt{\frac{9\cdot 8}{7}} = 3\cdot\sqrt{\frac{8}{7}}$ ✓

$\underline{L = \{-\frac{10}{7}\}}$

1 e) $x + \sqrt{x} = 42 \qquad D = [0, \infty)$

$\sqrt{x} = 42 - x \quad | (\,)^2$

$x = 1764 - 84x + x^2$

$x^2 - 85x + 1764 = 0$

$x_{1/2} = \frac{85 \pm \sqrt{7225 - 4 \cdot 1764}}{2} = \frac{85 \pm \sqrt{169}}{2} = \frac{85 \pm 13}{2}$

$x_1 = 49 \in D, \quad x_2 = 36 \in D$

Probe: $x_1 = 49$: $\quad 49 + \sqrt{49} = 49 + 7 \neq 42 \quad ↯$

$x_2 = 36$: $\quad 36 + \sqrt{36} = 36 + 6 = 42 \quad ✓ \qquad L = \{36\}$

2 a) $\sqrt{x-7} = 7 - \sqrt{x}$ Definitionsmenge: $x - 7 \geq 0$, $x \geq 7$; $\quad x \geq 0$

$x - 7 = 49 - 14\sqrt{x} + x \qquad D = [7; \infty)$

$-56 = -14\sqrt{x}$

$4 = \sqrt{x} \quad | (\,)^2$

$16 = x \in D$

Probe: $\sqrt{16-7} = 3 = 7 - \sqrt{16} \quad ✓ \qquad L = \{16\}$

b) $\sqrt{x+7} + \sqrt{x-1} = 4$ Definitionsmenge: $x + 7 \geq 0$, $x \geq -7$; $\quad x - 1 \geq 0$, $x \geq 1$

$\sqrt{x+7} = 4 - \sqrt{x-1} \quad | (\,)^2 \qquad D = [1; \infty)$

$x + 7 = 16 - 8\sqrt{x-1} + x - 1$

$8 = 16 - 8\sqrt{x-1}$

$-8 = -8\sqrt{x-1} \quad | : -8$

$1 = \sqrt{x-1} \quad | (\,)^2$

$1 = x - 1$

$x = 2 \in D \qquad$ Probe: $\sqrt{9} + \sqrt{1} = 4 \; ✓ \qquad L = \{2\}$

c) $\sqrt{x+1} = \sqrt{33-x} - 2$ Definitionsmenge: $x + 1 \geq 0$, $x \geq -1$; $\quad 33 - x \geq 0$, $33 \geq x$

$x + 1 = 33 - x - 4\sqrt{33-x} + 4 \qquad D = [-1; 33]$

$2x - 36 = -4\sqrt{33-x} \quad | : -2$

$18 - x = 2\sqrt{33-x} \quad | (\,)^2$

$324 - 36x + x^2 = 4(33 - x)$

$x^2 - 32x + 192 = 0$

$x_{1/2} = \frac{32 \pm \sqrt{1024 - 768}}{2} = \frac{32 \pm \sqrt{256}}{2} = \frac{32 \pm 16}{2}$

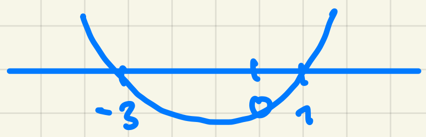
$x_1 = 24 \in D, \quad x_2 = 8 \in D$

Probe: $x_1 = 24$: $\quad \sqrt{25} = 5 \neq \sqrt{9} - 2 \quad ↯$

$x_2 = 8$: $\quad \sqrt{9} = \sqrt{25} - 2 \quad ✓ \qquad L = \{8\}$

(3) $\sqrt{x^2+2x-3} = \sqrt{x^2-8} - 1$

Definitionsmenge: (i) $x^2+2x-3 = 0$



Vieta : $x_1 = -3, \; x_2 = 1$

$x^2+2x-3 \geq 0$ für $x \in (-\infty, -3] \cup [1, \infty)$

(ii) $x^2 - 8 = 0$

$x^2 = 8 \qquad x_{1,2} = \pm\sqrt{8} = \pm 2\sqrt{2}$

$D = (-\infty; -3] \cup [2\sqrt{2}; \infty)$

$$\sqrt{x^2+2x-3} = \sqrt{x^2-8} - 1 \quad |()^2$$
$$x^2+2x-3 = x^2-8 - 2\sqrt{x^2-8} + 1$$
$$2x + 4 = -2\sqrt{x^2-8} \quad |:-2$$
$$-x-2 = \sqrt{x^2-8} \quad |()^2$$
$$x^2+4x+4 = x^2-8$$
$$4x = -12$$
$$x = -3 \in D$$

Probe: $\sqrt{9-6-3} = \sqrt{9-8} - 1$ ✓

$\underline{L = \{-3\}}$

(4) $\sqrt{6+x} + \sqrt{4+x} = \sqrt{4-x} + \sqrt{6-x}$

Definitionsmenge: $6+x \geq 0$, $x \geq -6$ $\quad 4+x \geq 0$, $x \geq -4$

$6-x \geq 0$, $6 \geq x$ $\quad 4-x \geq 0$, $4 \geq x$

$D = [-4; 4]$

$$\sqrt{6+x} - \sqrt{6-x} = \sqrt{4-x} - \sqrt{4+x} \quad |()^2$$
$$(6+x) - 2\sqrt{36-x^2} + (6-x) = (4-x) - 2\sqrt{16-x^2} + (4+x)$$
$$12 - 2\sqrt{36-x^2} = 8 - 2\sqrt{16-x^2} \quad |-8, :2$$
$$2 - \sqrt{36-x^2} = -\sqrt{16-x^2} \quad |()^2$$
$$4 - 4\sqrt{36-x^2} + (36-x^2) = 16-x^2$$
$$-4\sqrt{36-x^2} = -24 \quad |:-4$$
$$\sqrt{36-x^2} = 6 \quad |()^2$$
$$36 - x^2 = 36$$
$$x = 0 \in D$$

Probe: $\sqrt{6} + \sqrt{4} = \sqrt{4} + \sqrt{6}$ ✓

$\underline{L = \{0\}}$

(5) $\sqrt{x+3} = t+4$

Definitionsmenge: $x+3 \geq 0$, $x \geq -3$

$D = [-3, \infty)$

Es muss gelten : $t+4 \geq 0$, also $t \geq -4$

$$x+3 = t^2+8t+16$$
$$x = t^2+8t+13$$

Gibt es Werte von $t \geq -4$, so dass x nicht im Definitionsbereich liegt ? Nein, denn

$t^2+8t+16 = (t+4)^2 \geq 0$

$t^2+8t+16 = 0$

Vieta : $(t+4)^2 = 0$

$\underline{\text{Lösung: } x = t^2+8t+13 \text{ für } t \geq -4}$