(A1) a) $|x+2| = |x-3|$

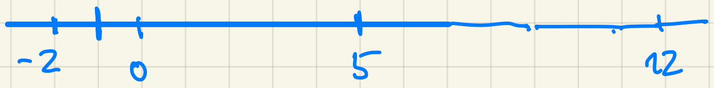
Welche Zahl hat zu den Zahlen -2 und 3 denselben Abstand? $\quad L = \{\frac{1}{2}\}$

$|x+2| \leq |x-3| \qquad L = (-\infty; \frac{1}{2}]$

b) $|5-2x| = 7$

$L = \{-1; 6\}$

Für welche Zahlen gilt, dass ihr Doppeltes zur Zahl 5 den Abstand 7 hat.



$|5-x| > 7$

$L = (-\infty, -1) \cup (6; +\infty)$

Für welche Zahlen hat ihr Doppeltes zur 5 den Abstand größer 7

c) $|x| = |x-5| \qquad L = \{\frac{5}{2}\}$

$|x| > |x-5| \qquad L = (\frac{5}{2}; +\infty)$

d) $|2x-4| = x \qquad L = \{4; \frac{4}{3}\}$ $\quad$ (Die beiden Fälle im Kopf)

$|2x-4| < x \qquad L = (\frac{4}{3}; 4)$

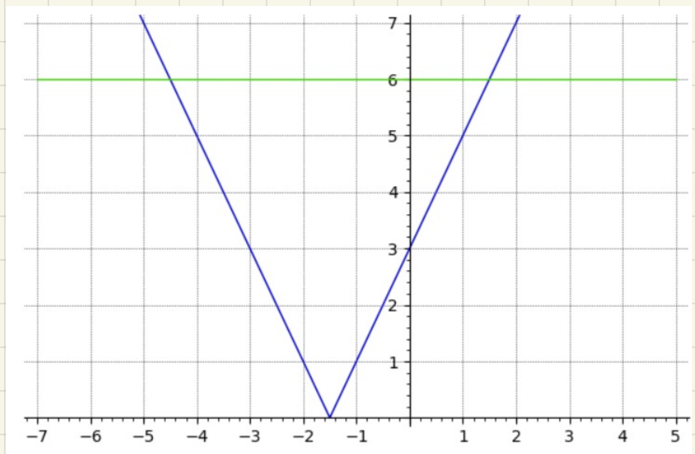
(2) a) $|2x+3| = 6$

$L = \{-4.5; 1.5\}$ $\quad$ (Abstands-überlegung)

(oder: $2|x+\frac{3}{2}| = 6$
$|x+\frac{3}{2}| = 3$) $\quad$ oder

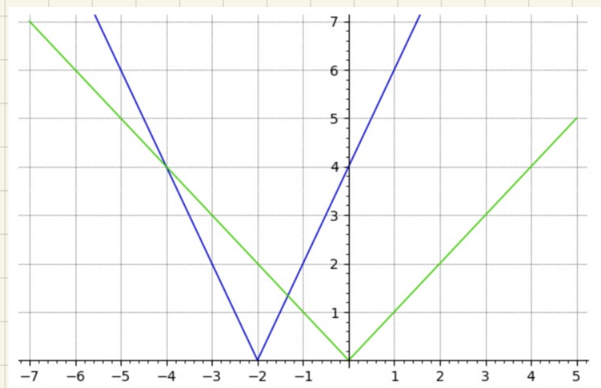
$|2x+3| \geq 6$ $\quad$ (globales Verhalten)

$L = (-\infty; -4.5) \cup (1.5; +\infty)$



b) $|2x+4| = |x|$, kritische Punkte: $x_1 = -2$, $x_2 = 0$

$I_1 = (-\infty, -2]$, $I_2 = (-2, 0]$, $I_3 = (0, \infty)$



$x \in I_1$:	$x \in I_2$:	$x \in I_3$:
$-(2x+4) = -x$	$2x+4 = -x$	$2x+4 = x$
$2x+4 = x$	$3x = -4$	$x = -4 \notin I_3$
$x = -4$	$x = -\frac{4}{3}$	
$L_1 = \{-4\}$	$L_2 = \{-\frac{4}{3}\}$	$L_3 = \{\}$

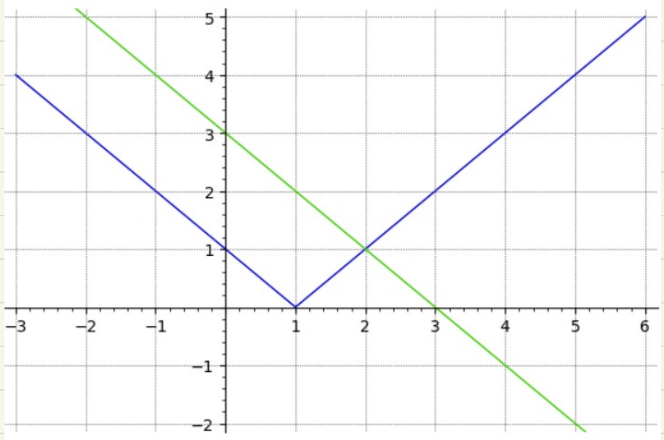
$\underline{L = \{-4; -\frac{4}{3}\}}$

Die Ungleichung:

$-(2x+4) < -x$	$2x+4 < -x$	$2x+4 < x$
$2x+4 > x$	$3x < -4$	$x < -4$
$x > -4$	$x < -4/3$	
$L_1 = (-4; -2]$	$L_2 = (-2; -\frac{4}{3})$	$L_3 = \{\}$

$L = (-4; -\frac{4}{3})$

② c) $|x-1| = 3-x$



Aus der Zeichnung: $L = \{2\}$

$|x-1| > 3-x$: $L = (2, +\infty)$

kritische Punkt: $x_1 = 1$

$I_1 = (-\infty, 1]$, $I_2 = (1, \infty)$

$x \in I_1$:
$$1-x = 3-x$$
$$1 = 3$$
$$L_1 = \{\ \}$$

$x \in I_2$
$$x-1 = 3-x$$
$$2x = 2$$
$$x = 1$$
$$L_2 = \{1\}$$

$\underline{L = \{1\}}$

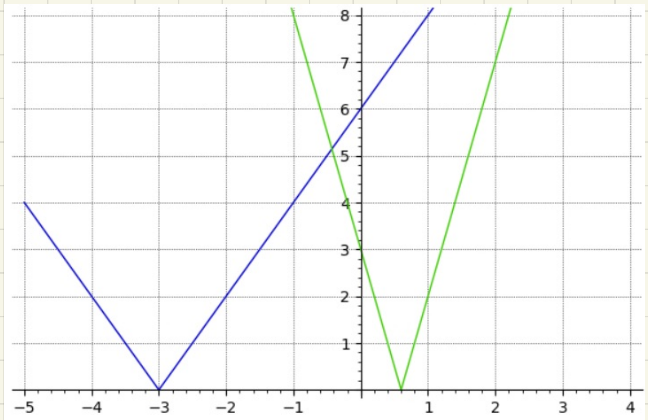
Die Ungleichung: $|x-1| > 3-x$

$$1-x > 3-x$$
$$1 > 3$$
$$L_1 = \{\ \}$$

$$x-1 > 3-x$$
$$2x > 4$$
$$x > 2$$

$L_2 = (2, \infty)$ $\quad \underline{L = (2, \infty)}$

d) $|2x+6| = |3-5x|$



kritische Punkte: $x_1 = -3$, $x_2 = \frac{3}{5}$

$I_1 = (-\infty; -3]$, $I_2 = (-3, \frac{3}{5}]$, $I_3 = (\frac{3}{5}, \infty)$

$x \in I_1$:
$$-(2x+6) = 3-5x$$
$$-2x-6 = 3-5x$$
$$3x = 9$$
$$x = 3 \notin I_1$$
$$L_1 = \{\ \}$$

$x \in I_2$:
$$2x+6 = 3-5x$$
$$7x = -3$$
$$x = -3/7$$
$$L_2 = \{-3/7\}$$

$x \in I_3$:
$$2x+6 = 5x-3$$
$$-3x = -9$$
$$x = 3$$
$$L_3 = \{3\}$$

$\underline{L = \{-3/7; 3\}}$

Die Ungleichung: $|2x+6| \leq |3-5x|$

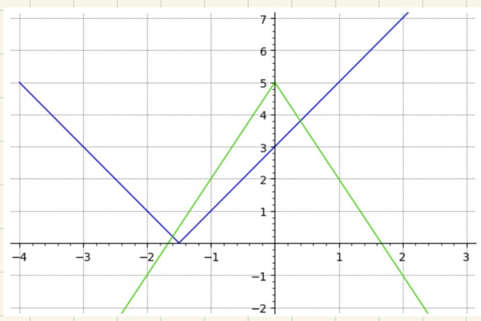
$$-(2x+6) \leq 3-5x$$
$$-2x-6 \leq 3-5x$$
$$3x \leq 9$$
$$x \leq 3$$
$$L_1 = (-\infty, -3)$$

$$2x+6 \leq 3-5x$$
$$7x \leq -3$$
$$x \leq -3/7$$
$$L_2 = (-3; -3/7]$$

$$2x+6 \leq 5x-3$$
$$-3x \leq -9$$
$$x \geq 3$$
$$L_3\ [3, \infty)$$

$L = (-\infty, \frac{3}{7}] \cup [3; \infty)$

e) $|3+2x| = 5-3\cdot|x|$



kritische Punkte: $x_1 = -\frac{3}{2}$, $x_2 = 0$

$I_1 = (-\infty, -\frac{3}{2}]$, $I_2 = (-\frac{3}{2}, 0]$, $I_3 = (0, \infty)$

$x \in I_1$:
$$-3-2x = 5-3(-x)$$
$$-5x = 8$$
$$x = -8/5$$
$$L_1 = \{-8/5\}$$

$x \in I_2$:
$$3+2x = 5+3x$$
$$-x = 2$$
$$x = -2$$
$$L_2 = \{\ \}$$

$x \in I_3$:
$$3+2x = 5-3x$$
$$5x = 2$$
$$x = 2/5$$
$$L_3 = \{2/5\}$$

$\underline{L = \{-\frac{8}{5}; \frac{2}{5}\}}$

Die Ungleichung $|3+2x| \geq 5-3\cdot|x|$

$$-3-2x \geq 5+3x$$
$$-5x \geq 8$$
$$x \leq -8/5$$

$$3+2x \geq 5+3x$$
$$-x \geq 2$$
$$x \leq -2$$

$$3+2x \geq 5-3x$$
$$5x \geq 2$$
$$x \geq 2/5$$

$L_1 = (-\infty, -\frac{8}{5})$ $\quad L_2 = \{\ \}$ $\quad L_3 = (\frac{2}{5}, \infty)$ $\quad \underline{L = (-\infty; -\frac{8}{5}) \cup (\frac{2}{5}; \infty)}$

③ a) $|x-2| + |4-x| \leq x+1$

kritische Punkte: $x_1 = 2$, $x_2 = 4$

1.Fall: $x \in (-\infty;2]$

$-(x-2)+(4-x) \leq x+1$

$-x+2+4-x \leq x+1$

$5 \leq 3x$

$\frac{5}{3} \leq x$

$L_1 = [\frac{5}{3}; 2]$

2. Fall: $x \in (2;4]$

$x-2+4-x \leq x+1$

$1 \leq x$

$L_2 = (2;4]$

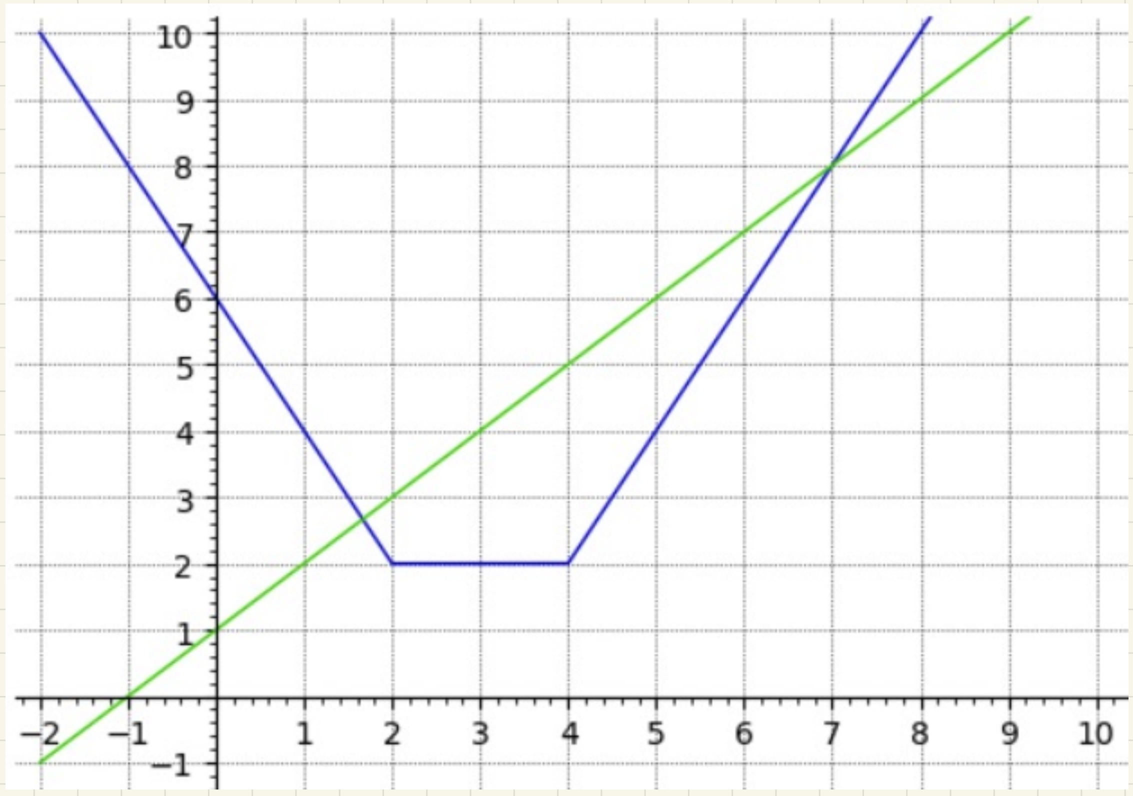
3. Fall: $x \in (4,+\infty)$

$x-2-4+x \leq x+1$

$x \leq 7$

$L_3 = (4;7]$

$\underline{L = (\frac{5}{3}; 7]}$



b) $2 \cdot |1-3x| > 2 + |3x+8|$

kritische Punkte: $x_1 = \frac{1}{3}$, $x_2 = -\frac{8}{3}$

1.Fall: $x \in (-\infty, -\frac{8}{3}]$

$2(1-3x) > 2 - 3x - 8$

$2-6x > 2-3x-8$

$8 > 3x$

$\frac{8}{3} > x$

$L_1 = (-\infty; -\frac{8}{3}]$

2. Fall $x \in (-\frac{8}{3}; \frac{1}{3}]$

$2(1-3x) > 2+3x+8$

$2-6x > 2+3x+8$

$-8 > 9x$

$-\frac{8}{9} > x$

$L_2 = (-\frac{8}{3}; -\frac{8}{9})$

3. Fall: $x \in (\frac{1}{3}; \infty)$

$2(3x-1) > 2+3x+8$

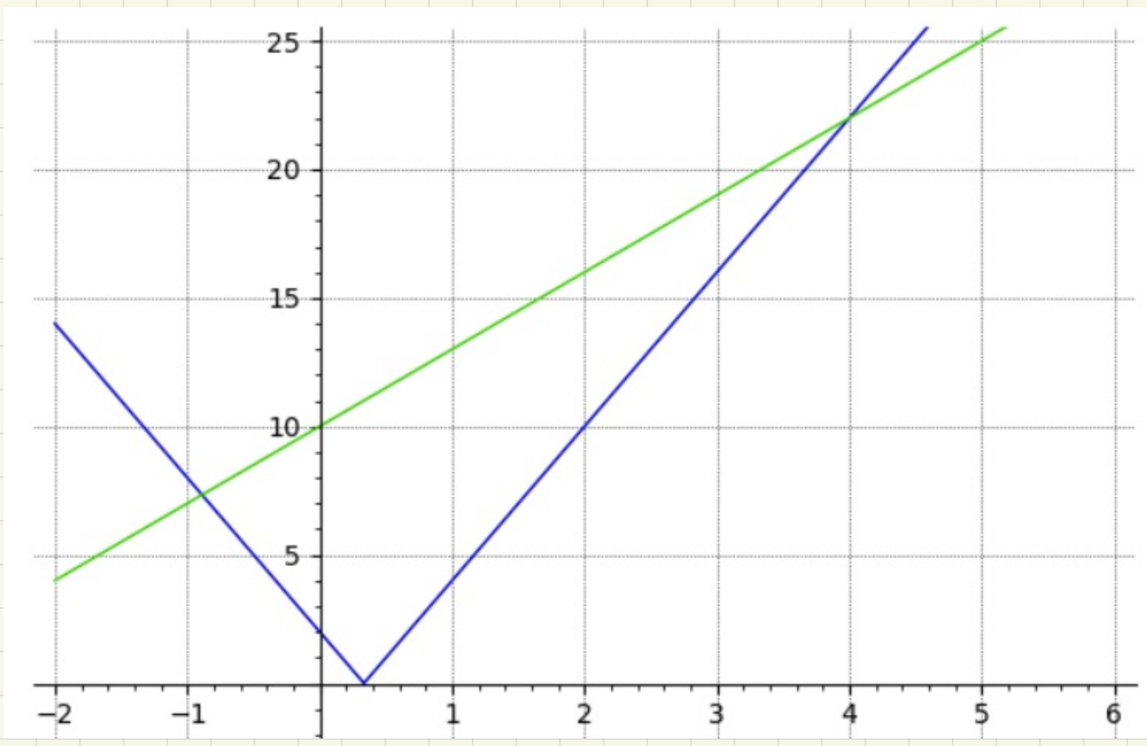
$6x-2 > 2+3x+8$

$3x > 12$

$x > 4$

$L_3 = (4;\infty)$

$L = (-\infty; -\frac{8}{9}) \cup (4;\infty)$



③ c) $|x-6| < 2x - |4-3x|$

Kritische Punkte $x_1 = 6$, $x_2 = \frac{4}{3}$

1. Fall: $x \in (-\infty; \frac{4}{3}]$

$6-x < 2x-(4-3x)$

$6-x < 2x-4+3x$

$10 < 6x$

$\frac{5}{3} < x$

$L_1 = \{\}$

2. Fall: $x \in (\frac{4}{3}; 6]$

$6-x < 2x-(3x-4)$

$6-x < 2x-3x+4$

$2 < 0$

$L_2 = \{\}$

3. Fall: $x \in (6; +\infty)$

$x-6 < 2x-(3x-4)$

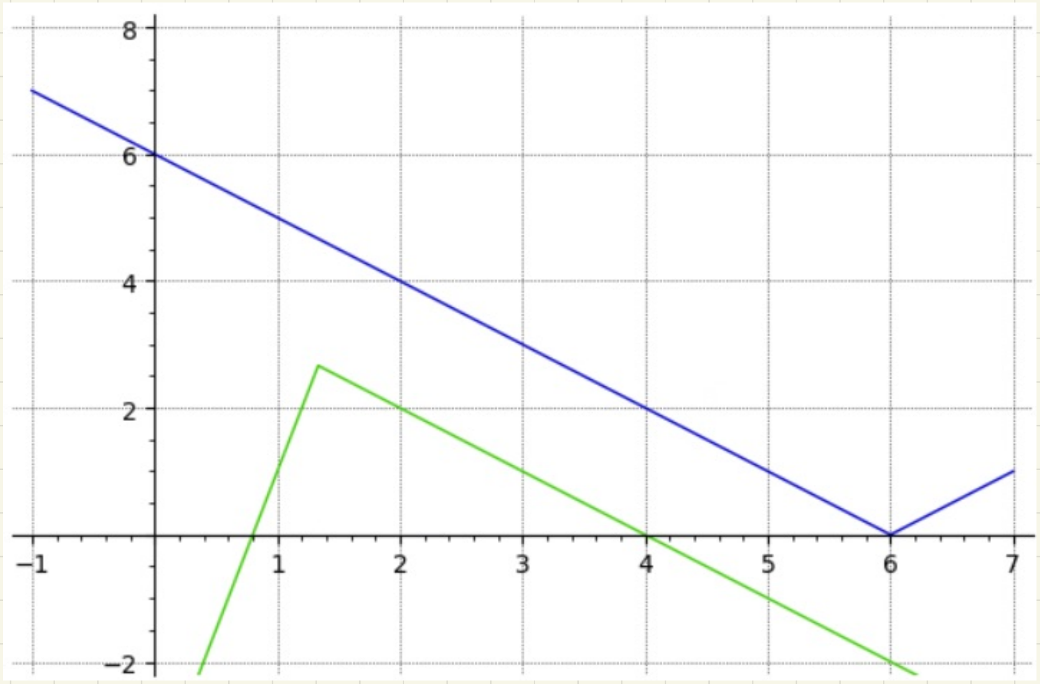
$x-6 < 2x-3x+4$

$2x < 10$

$x < 5$

$L_3 = \{\}$

$\underline{L = \{\}}$



d) $|3x+6| + 2x \ge 8 - 3\cdot|3x+2|$

Kritische Punkte: $x_1 = -\frac{6}{3} = -2$, $x_2 = -\frac{2}{3}$

1. Fall: $x \in (-\infty; -2]$

$-3x-6+2x \ge 8+3\cdot(3x+2)$

$-x-6 \ge 8+9x+6$

$-20 \ge 10x$

$-2 \ge x$

$L_1 = (-\infty; -2]$

2. Fall: $x \in (-2; -\frac{2}{3}]$

$3x+6+2x \ge 8+3\cdot(3x+2)$

$5x+6 \ge 8+9x+6$

$-8 \ge 4x$

$-2 \ge x$

$L_2 = \{\}$

3. Fall: $x \in (-\frac{2}{3}; +\infty)$

$3x+6+2x \ge 8-3(3x+2)$

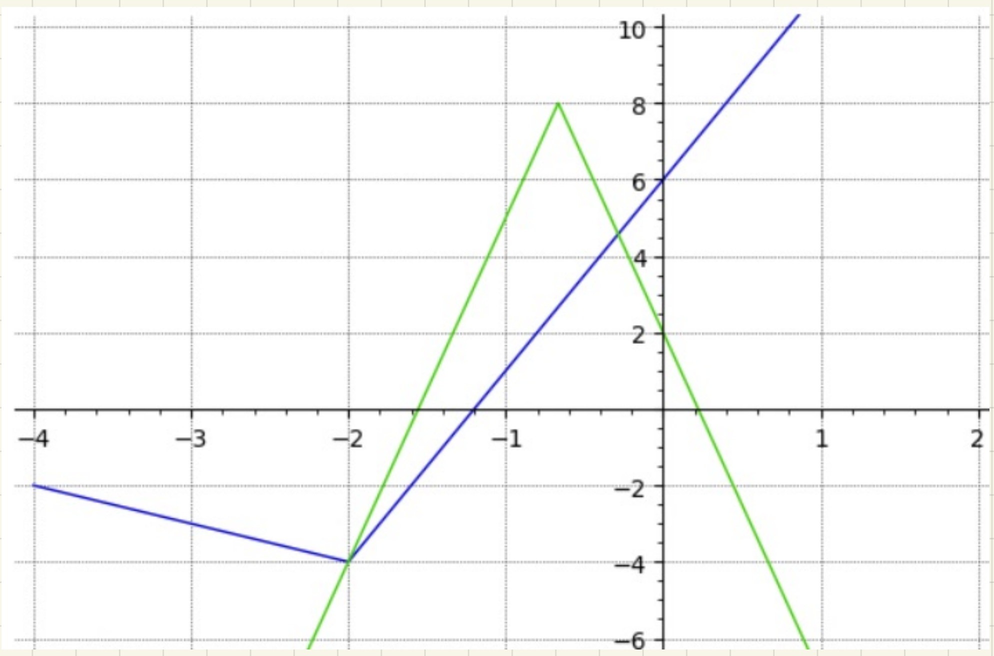
$5x+6 \ge 8-9x-6$

$14x \ge -4$

$x \ge -\frac{2}{7}$

$L_3 = [-\frac{2}{7}; +\infty)$

$\underline{L = (-\infty; -2] \cup [-\frac{2}{7}; +\infty)}$



(4) $|2x-5| + x = a - 3x$

$|2x-5| = a - 4x$

1. Fall: $x \in [-\infty; \frac{5}{2}]$

$5 - 2x = a - 4x$

$2x = a - 5$

$x = \frac{a-5}{2}$

Es muss gelten: $\frac{a-5}{2} \leq \frac{5}{2}$

$a \leq 10$

Also: $x = \frac{a-5}{2}$ falls $a \leq 10$

2. Fall: $x \in (\frac{5}{2}; \infty)$

$2x - 5 = a - 4x$

$6x = a + 5$

$x = \frac{a+5}{6}$

Es muss gelten: $\frac{a+5}{6} > \frac{5}{2}$

$a + 5 > 15$

$a > 10$

Also $x = \frac{a+5}{6}$ falls $a > 10$

<u>Lösung:</u>

$$x = \begin{cases} \frac{a-5}{2} & \text{falls } a \leq 10 \\ \frac{a+5}{6} & \text{falls } a > 10 \end{cases}$$