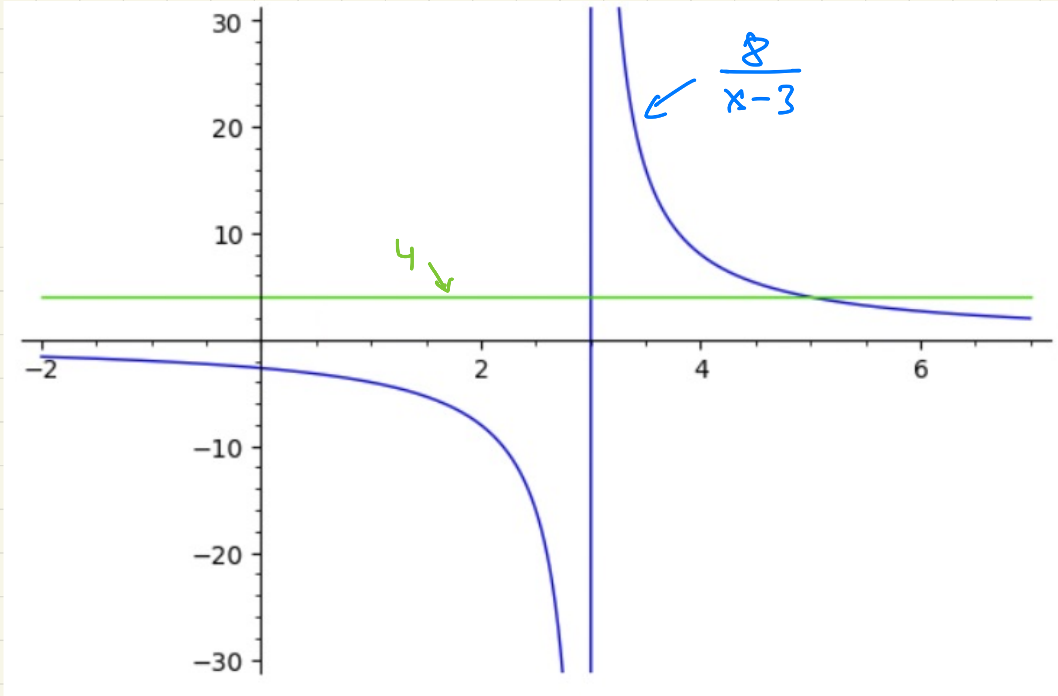
A1 a) $\frac{8}{x-3} < 4$ $\qquad D = \mathbb{R} \setminus \{3\}$



1. Fall $x - 3 > 0$

$x > 3$:

$$8 < 4(x-3) = 4x - 12$$
$$20 < 4x$$
$$x > 5 \qquad L_1 = (5, +\infty)$$

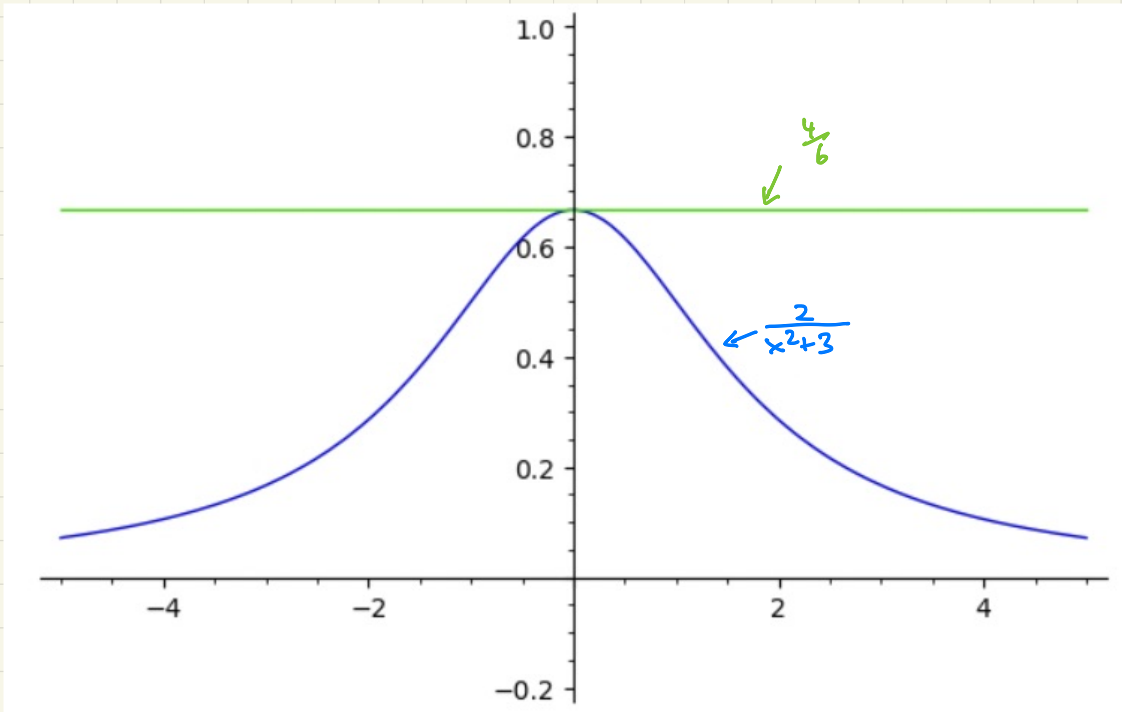
2. Fall $x < 3$:

$$8 > 4x - 12$$
$$x < 5 \qquad L_2 = (-\infty, 3)$$

$$L = (-\infty, 3) \cup (5, +\infty)$$

b) $\frac{4}{6} \geq \frac{2}{x^2+3}$ $\quad \left| \cdot (x^2+3) \right.$ $\quad D = \mathbb{R}$

> 0 für $x \in \mathbb{R}$



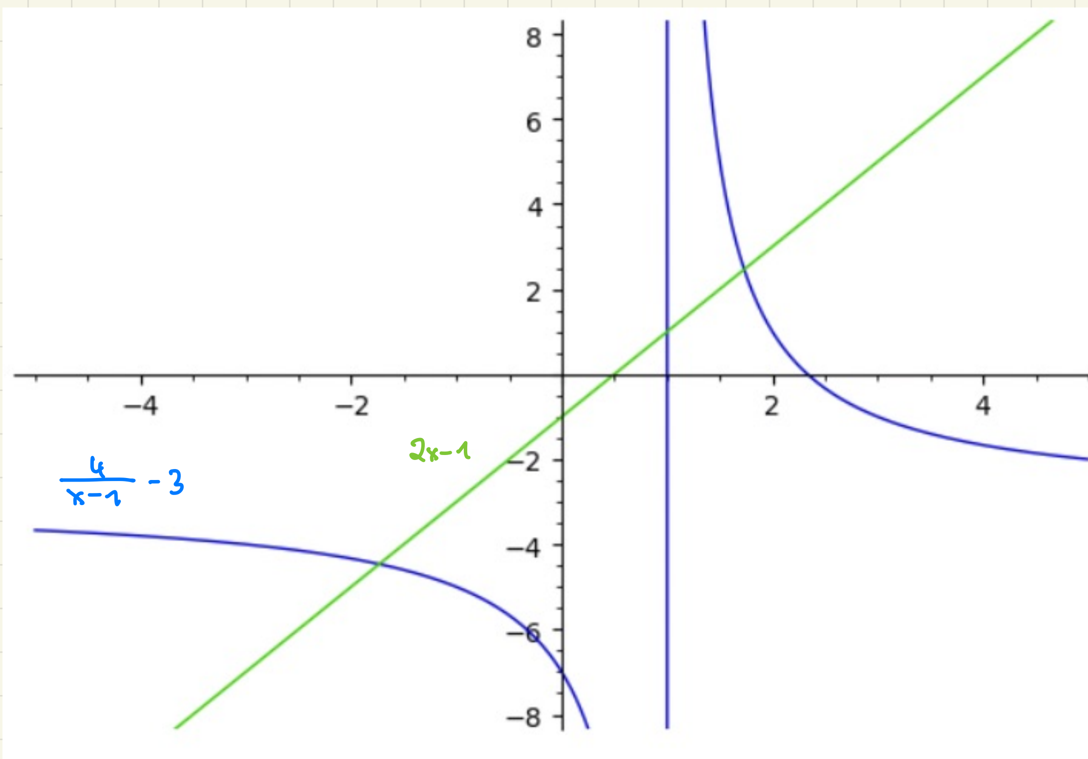
$$\frac{4}{6}(x^2+3) \geq 2$$
$$\frac{2}{3}x^2 \geq 2 - 2 = 0$$
$$x^2 \geq 0 \qquad L = \mathbb{R}$$

c) $\frac{4}{x-1} - 3 > 2x - 1$ $\qquad D = \mathbb{R} \setminus \{1\}$

1. Fall $x - 1 > 0$

$x > 1$

$$4 - 3(x-1) > (2x-1)(x-1) = 2x^2 - 2x - x + 1$$
$$4 - 3x + 3 > 2x^2 - 3x + 1$$
$$6 > 2x^2$$
$$x^2 < 3 \qquad L_1 = (1; \sqrt{3})$$



2. Fall: $x < 1$:

$\vdots$

$$x^2 > 3$$
$$|x| > \sqrt{3} \qquad L_2 = (-\infty, -\sqrt{3})$$

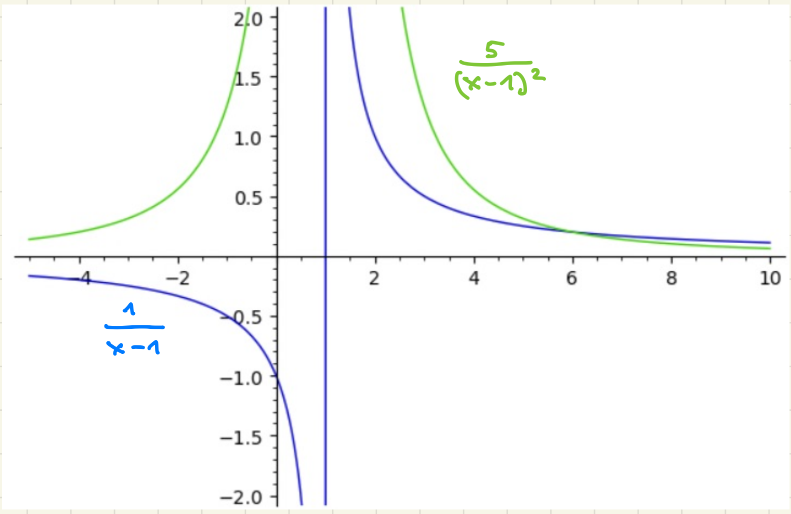
$$L = (-\infty; -\sqrt{3}) \cup (1; \sqrt{3})$$

② a) $\frac{1}{x-1} \geq \frac{5}{(x-1)^2} \quad \Big| \cdot (x-1)^2 \qquad D = \mathbb{R} \setminus \{1\}$

> 0 für alle $x \in D$

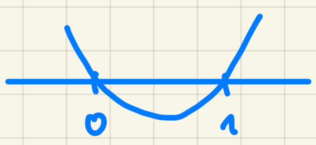
$$x-1 \geq 5$$
$$x \geq 6$$

$$\underline{L = [6, +\infty)}$$



b) $\frac{x}{x-1} > \frac{1}{x} \quad \Big| \, (x-1)\cdot x \qquad D = \mathbb{R} \setminus \{0; 1\}$

1. Fall: $x \in (-\infty, 0) \cup (1, \infty)$



$$x^2 > x-1$$
$$x^2 - x + 1 > 0$$
$$x^2 - x + 1 = 0 \qquad x_{1/2} = \frac{1 \pm \sqrt{\underbrace{1-4\cdot 1}_{<0}}}{2}$$

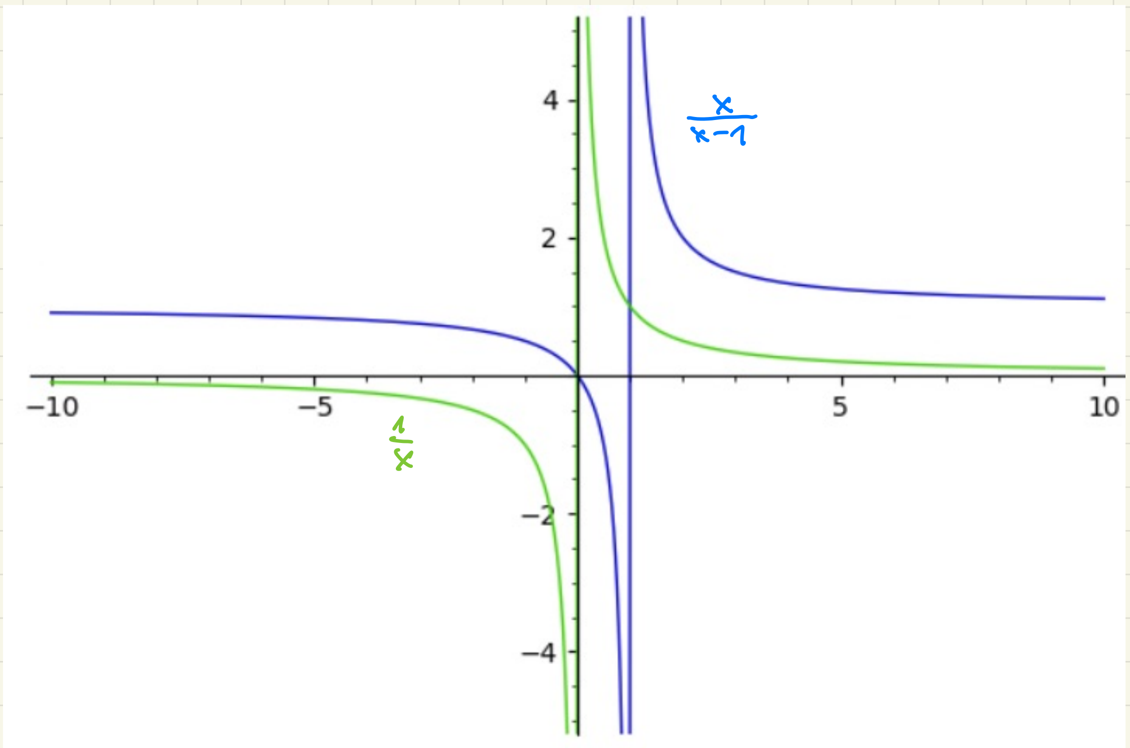
Keine Nullstellen, d.h. $x > 0$ überall

$\Rightarrow L_1 = (-\infty, 0) \cup (1, \infty)$

2. Fall: $x \in (0, 1)$

$$x^2 < x-1$$
$$x^2 - x + 1 < 0 \qquad L_2 = \{\ \}$$

$$\underline{L = (-\infty, 0) \cup (1, \infty)}$$



② c) $\frac{x}{x+3} < \frac{5x+1}{2x} \quad | \, 2x(x+3) \qquad D = \mathbb{R} \setminus \{-3; 0\}$

1.Fall $x \in (-\infty, -3) \cup (0, +\infty)$

$2x^2 < (5x+1)(x+3) = 5x^2 + 15x + x + 3$

$0 < 3x^2 + 16x + 3$

$0 = 3x^2 + 16x + 3 \quad , \quad x_{1/2} = \frac{-16 \pm \sqrt{256 - 36}}{6} = \frac{-16 \pm \sqrt{220}}{6} = -\frac{8}{3} \pm \frac{1}{3}\sqrt{55}$

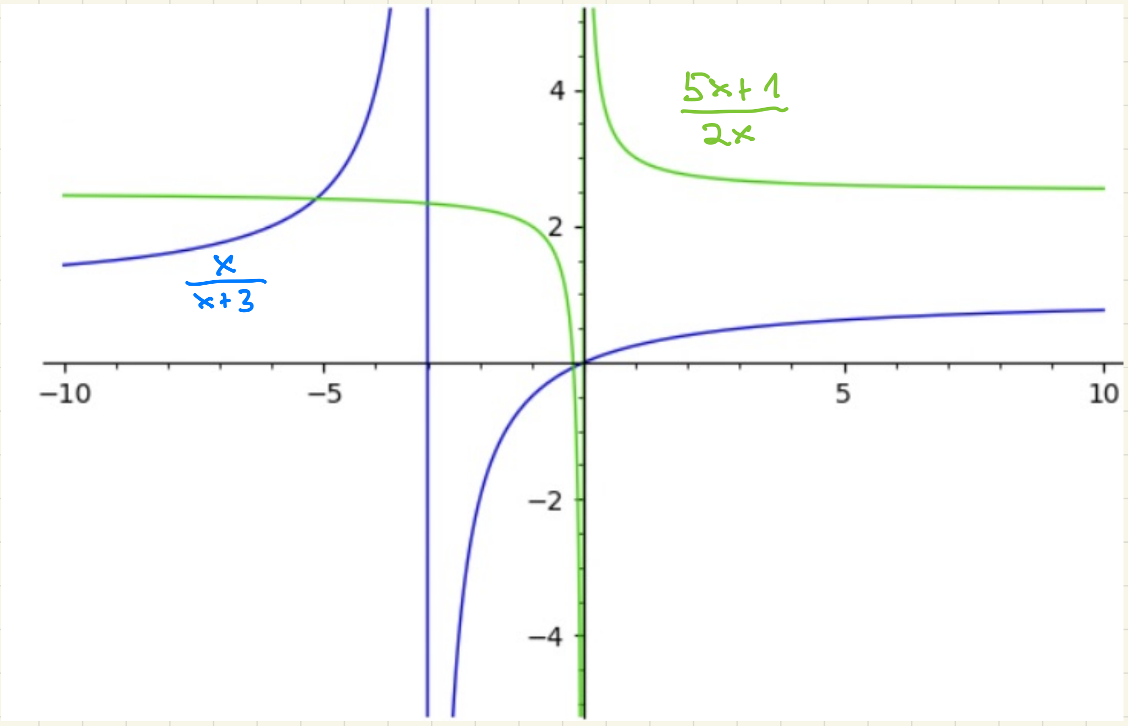
$x_1 \approx -5.14 \quad , \quad x_2 \approx -0.19$

$\Rightarrow L_1 = (-\infty, -\frac{8}{3} - \frac{1}{3}\sqrt{55}) \cup (0, +\infty)$

2.Fall: $x \in (-3, 0)$

$0 > 3x^2 + 16x + 13 \qquad L_2 = (-3, -\frac{3}{8} + \frac{1}{3}\sqrt{55})$

$L = (-\infty, -\frac{8}{3} - \frac{1}{3}\sqrt{55}) \cup (-3, -\frac{8}{3} + \frac{1}{3}\sqrt{55}) \cup (0, +\infty)$



d) $\frac{2x}{x+4} \geq \frac{x+1}{x-2} \quad | \cdot (x+4)(x-2) \qquad D = \mathbb{R} \setminus \{-4; 2\}$

1.Fall: $x \in (-\infty; -4) \cup (2; +\infty)$

$2x(x-2) \geq (x+1)(x+4)$

$2x^2 - 4x \geq x^2 + 4x + x + 4 = x^2 + 5x + 4$

$x^2 - 9x - 4 \geq 0$

$x^2 - 9x - 4 = 0 \qquad x_{1/2} = \frac{9 \pm \sqrt{81 + 16}}{2} = \frac{9}{2} \pm \frac{1}{2}\sqrt{97}$

$x_1 \approx -0.42 , \quad x_2 \approx 9.42$

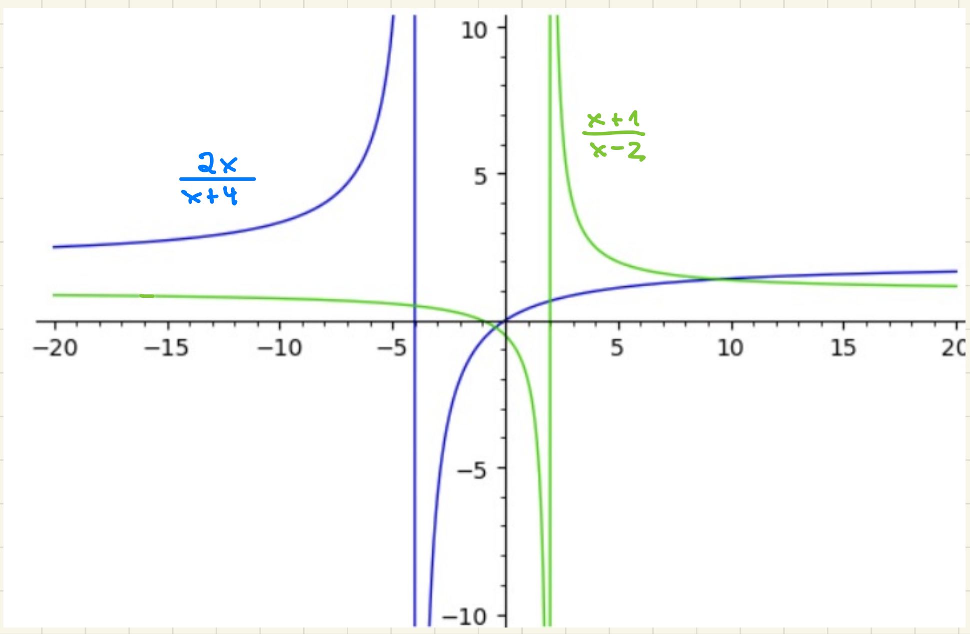
$L_1 = (-\infty, -4) \cup [\frac{9}{2} + \frac{1}{2}\sqrt{97}, \infty)$

2.Fall: $x \in (-4, 2)$

$x^2 - 9x - 4 \leq 0$

$L_2 = [\frac{9}{2} - \frac{1}{2}\sqrt{97}, 2)$

$L = (-\infty, -4) \cup [\frac{9}{2} - \frac{1}{2}\sqrt{97}, 2) \cup [\frac{9}{2} + \frac{1}{2}\sqrt{97}, \infty)$



② e) $\dfrac{3}{x+2} < \dfrac{6x}{x-1} \quad \Big| \,(x+2)(x-1)$ $\qquad D = \mathbb{R} \setminus \{-2; 1\}$

1.Fall : $x \in (-\infty; -2) \cup (1; \infty)$

$3(x-1) < 6x(x+2)$

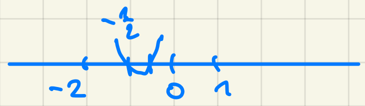
$3x - 3 < 6x^2 + 12x$

$0 < 6x^2 + 9x + 3 \quad | :3$

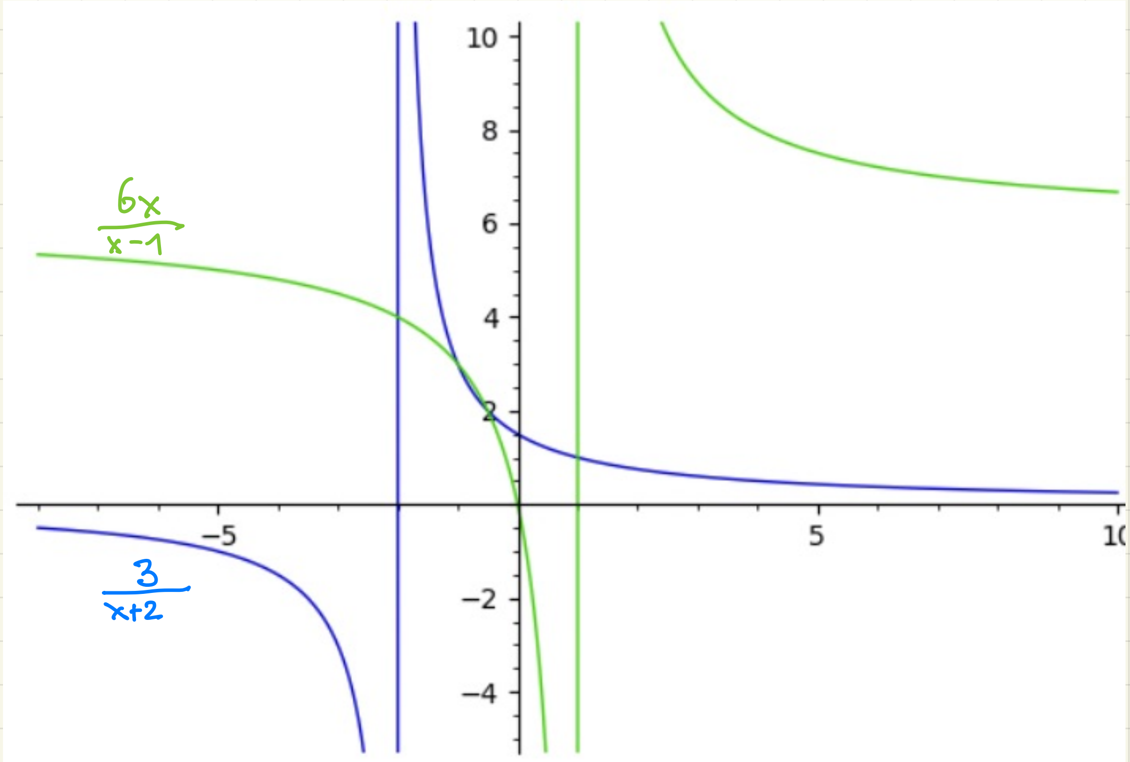
$0 < 2x^2 + 3x + 1$

$2x^2 + 3x + 1 = 0 \qquad x_{1/2} = \dfrac{-3 \pm \sqrt{9-8}}{4} = \dfrac{-3 \pm 1}{4}$

$x_1 = -\frac{1}{2}\,, \quad x_2 = -1$



$L_1 = (-\infty; -2) \cup (1; \infty)$



2. Fall : $x \in (-2; 1)$

$0 > 2x^2 + 3x + 1 \qquad L_2 = (-1; -\frac{1}{2})$

$L = (-\infty; -2) \cup (-1; -\frac{1}{2}) \cup (1; \infty)$

f) $\dfrac{4}{x+1} \geq \dfrac{2x}{x+3} \quad \cdot \Big| \,(x+1)(x+3)$

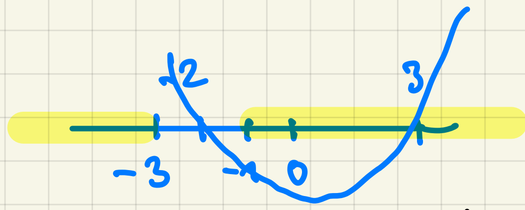
1. Fall $x \in (-\infty; -3) \cup (-1; \infty)$

$4(x+3) \geq 2x(x+1)$

$4x + 12 \geq 2x^2 + 2x$

$0 \geq 2x^2 - 2x - 12$

$0 \geq x^2 - x - 6$

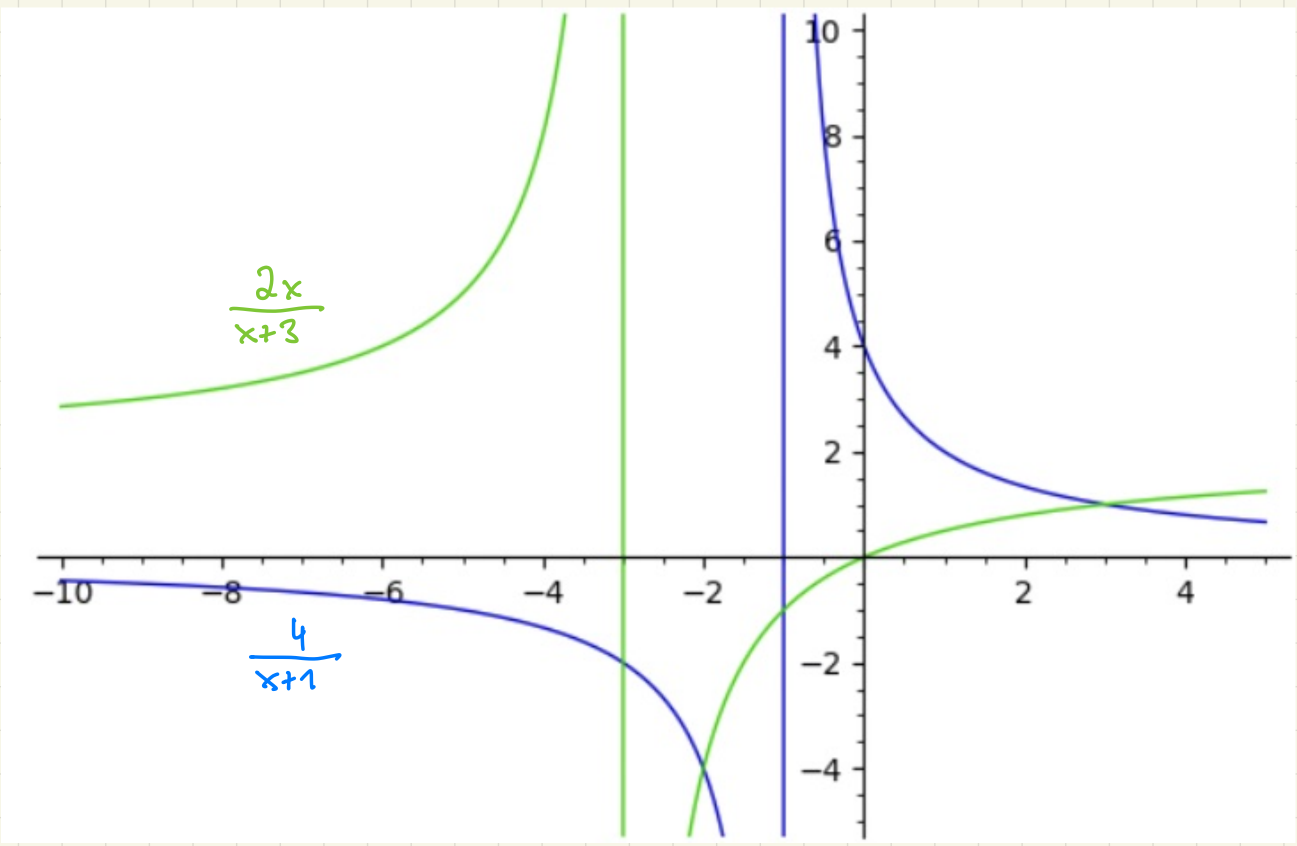


Vieta: $(x-3)(x+2) \leq 0 \qquad L_1 = [-1; 3]$

2. Fall : $x \in (-3, -1)$

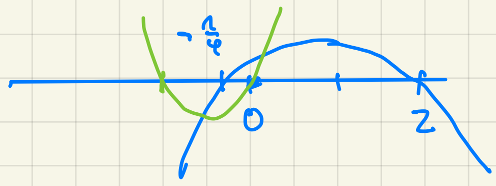
$0 \leq x^2 - x - 6 \qquad L_2 = (-3, -2]$

$L = (-3, -2] \cup [-1; 3]$



g) $\dfrac{2}{-x+2} < \dfrac{3x+1}{4x+1} \quad \Big| (-x+2)(4x+1) \qquad D = \mathbb{R} \setminus \{2; -\tfrac{1}{4}\}$

1. Fall: $x \in (-\tfrac{1}{4}; 2)$



$2(4x+1) < (3x+1)(-x+2)$

$8x+2 < -3x^2+6x-x+2$

$3x^2+3x < 0$

$3x(x+1) < 0 \qquad L_1 = (-\tfrac{1}{4}; 0)$

2. Fall: $x \in (-\infty; -\tfrac{1}{4}) \cup (2; \infty)$

$3x(x+1) > 0 \qquad L_2 = (-\infty; -1) \cup (2; \infty)$

$L = (-\infty; -1) \cup (-\tfrac{1}{4}; 0) \cup (2; \infty)$