A0

$(a+b)^2 = a^2 + 2ab + b^2$ Bsp: $(x+1)^2 = x^2 + 2x + 1$

$(3+2x)^2 = 9 + 12x + 4x^2$

$(a-b)^2 = a^2 - 2ab + b^2$ Bsp: $(x-2)^2 = x^2 - 4x + 4$

$(2x-\frac{1}{2})^2 = 4x^2 - 2x + \frac{1}{4}$

$(a+b)(a-b) = a^2 - b^2$ Bsp: $(x+1)(x-1) = x^2 - 1$

$(5y+4)(5y-4) = 25y^2 - 16$

A1 a) $\frac{3}{x} + 2 = x \quad | \cdot x$ $\quad D = \mathbb{R} \setminus \{0\}$

$3 + 2x = x^2 \quad | -3-2x$

$x^2 - 2x - 3 = 0$ Vieta: $x_1 = 3, x_2 = -1$

$L = \{-1; 3\}$

b) $\frac{1}{3x^2} - 1 = \frac{1}{6x} \quad | \cdot 6x^2$ $\quad D = \mathbb{R} \setminus \{0\}$

$2 - 6x^2 = x \quad | +6x^2 - 2$

$6x^2 + x - 2 = 0$

$x_{1/2} = \frac{-1 \pm \sqrt{1+48}}{12} = \frac{-1 \pm 7}{12}$

$x_1 = \frac{1}{2}, \quad x_2 = -\frac{2}{3}$

$L = \{\frac{1}{2}; -\frac{2}{3}\}$

c) $\frac{1}{x+2} + x = \frac{3x+7}{x+2} \quad | \cdot (x+2)$ $\quad D = \mathbb{R} \setminus \{-2\}$

$1 + x^2 + 2x = 3x + 7$

$x^2 - x - 6 = 0$

vieta $\Rightarrow x_1 = 3, x_2 = -2$ $\quad L = \{3\}$

d) $\frac{2x+1}{3} + \frac{10}{2x+1} = 4 \quad | \cdot 3(2x+1)$ $\quad D = \mathbb{R} \setminus \{-\frac{1}{2}\}$

$(2x+1)^2 + 30 = 12(2x+1)$

$4x^2 + 4x + 1 + 30 = 24x + 12$

$4x^2 - 20x + 19 = 0$

$96 = 4 \cdot 24 = 4 \cdot 4 \cdot 6$

$x_{1/2} = \frac{20 \pm \sqrt{400 - 304}}{8} = \frac{20 \pm \sqrt{96}}{8} = \frac{20 \pm 4\sqrt{6}}{8}$

$x_{1/2} = \frac{5}{2} \pm \frac{1}{2}\sqrt{6}$ $\quad L = \{\frac{5}{2} \pm \frac{1}{2}\sqrt{6}\}$

A2 a) $\frac{3}{x+1} + \frac{4}{x-2} = \frac{3}{2x+2} \quad | \cdot 2(x+1)(x-2)$ $\qquad D = \mathbb{R} \setminus \{-1; 2\}$

$6(x-2) + 8(x+1) = 3(x-2)$

$6x - 12 + 8x + 8 = 3x - 6$

$11x + 2 = 0$

$x = -\frac{2}{11}$ $\qquad L = \left\{-\frac{2}{11}\right\}$

b) $\frac{x}{x-1} + \frac{2}{x-2} = \frac{1}{x-1} + \frac{x}{x-2} \quad \cdot | (x-1)(x-2)$ $\qquad D = \mathbb{R} \setminus \{1; 2\}$

$x(x-2) + 2(x-1) = (x-2) + x(x-1)$

$x^2 - 2x + 2x - 2 = x - 2 + x^2 - x$

$0 = 0$ $\qquad L = D = \mathbb{R} \setminus \{1; 2\}$

ist wahr für alle $x \in D$

c) $\frac{x}{2x-3} - \frac{1}{2x} = \frac{3}{4x-6} \quad | (2x-3) \cdot 2 \cdot x$ $\qquad D = \mathbb{R} \setminus \{0; \frac{3}{2}\}$

$2x^2 - (2x-3) = 3x$

$2x^2 - 5x + 3 = 0$

$x_{1/2} = \frac{5 \pm \sqrt{25-24}}{4} = \frac{5 \pm 1}{4}$

$x_1 = \frac{3}{2}, \quad x_2 = 1$ $\qquad L = \{1\}$

d) $\frac{36}{x+6} - 36 = \frac{36}{x-6} \quad | (x-6)(x+6)$ $\qquad D = \mathbb{R} \setminus \{-6; 6\}$

$36(x-6) - 36(x^2-36) = 36(x+6) \quad | :36$

$x - 6 - (x^2 - 36) = x + 6$

$-x^2 + 36 - 12 = 0$

$x^2 = 24$

$x_{1/2} = \pm 2\sqrt{6}$ $\qquad L = \{\pm 2\sqrt{6}\}$

A3 a) $\frac{3}{x-4} - \frac{24}{x^2-16} = \frac{3}{x+4} - x^2 + 16 \quad | \cdot (x-4)(x+4)$ $\qquad D = \mathbb{R} \setminus \{-4; 4\}$

$3(x+4) - 24 = 3(x-4) - (x^2-16)(x^2-16)$

$3x + 12 - 24 = 3x - 12 - (x^2-16)^2$

$(x^2-16)^2 = 0$

$x^2 = 16$ $\qquad x_1 = 4, \; x_2 = -4$ $\qquad L = \{\}$

b) $\frac{7(x-5)^2}{6x^2-6} = \frac{5x-1}{3x+3} - \frac{3x-2}{6x-6} \quad | \cdot 6(x+1)(x-1)$ $\qquad D = \mathbb{R} \setminus \{-1; +1\}$

$6(x-1)(x+1) \qquad 3(x+1) \qquad 6(x-1)$

$7(x-5)^2 = 2(5x-1)(x-1) - (3x-2)(x+1)$

$7(x^2 - 10x + 25) = 10x^2 - 10x - 2x + 2 - (3x^2 + 3x - 2x - 2)$

$7x^2 - 70x + 175 = 10x^2 - 12x + 2 - 3x^2 - x + 2$

$-57x = -171$

$x = 3$ $\qquad L = \{3\}$

A3 c) $\frac{3x+2}{x-2} = \frac{x+2}{3x-2} \quad | \cdot (x-2)(3x-2)$ $\quad D = \mathbb{R} \setminus \{2; \frac{2}{3}\}$

$(3x+2)(3x-2) = (x+2)(x-2)$

$9x^2 - 4 = x^2 - 4$

$8x^2 = 0$

$x = 0 \qquad L = \{0\}$

d) $\frac{5x+1}{x+2} = 3 + \frac{2x^2+3x-8}{x^2+4x+4} \quad | \cdot (x+2)^2$ $\quad D = \mathbb{R} \setminus \{-2\}$

$(5x+1)(x+2) = 3(x+2)^2 + 2x^2 + 3x - 8$

$5x^2 + 10x + x + 2 = 3x^2 + 12x + 12 + 2x^2 + 3x - 8$

$-4x = 2$

$x = -\frac{1}{2} \qquad L = \{-\frac{1}{2}\}$

4 a) $\frac{x}{a} - \frac{a}{x} = \frac{3}{2} \quad | \cdot ax$ $\quad D = \mathbb{R} \setminus \{0\}, \ a \neq 0$

$x^2 - a^2 = \frac{3}{2}ax$

$x^2 - \frac{3}{2}ax - a^2 = 0$

$x_{1/2} = \frac{\frac{3}{2}a \pm \sqrt{\frac{9}{4}a^2 + 4a^2}}{2} = \frac{3}{4}a \pm \frac{|a|}{2}\sqrt{\frac{25}{4}} = \frac{3}{4}a \pm \frac{5}{4}|a|$

Falls $a > 0$: $x_{1/2} = \frac{3}{4}a \pm \frac{5}{4}a$

Falls $a < 0$: $x_{1/2} = \frac{3}{4}a \mp \frac{5}{4}a$

$L = \{2a; -\frac{1}{2}a\}$

b) $\frac{x+a}{x-a} - \frac{x-a}{x+a} = \frac{8a^2}{x^2-a^2} \quad | \cdot (x^2-a^2)$ $\quad D = \mathbb{R} \setminus \{-a; +a\}$

$(x+a)^2 - (x-a)^2 = 8a^2$

$x^2 + 2ax + a^2 - (x^2 - 2ax + a^2) = 8a^2$

$4ax = 8a^2$

1. Fall $a = 0$: $L = \mathbb{R} \setminus \{0\}$

2. Fall $a \neq 0$: $x = 2a$, $L = \{2a\}$