Bestimme die Grenzwerte der Folge (a_n)

a) $a_n = \frac{5}{n+4}$

b) $a_n = \frac{2n}{3n-2}$

c) $a_n = \frac{6n+1}{2n-1}$

d) $a_n = \frac{2\cdot 3^n + 1}{3^{n-1} - 2}$

e) $a_n = \frac{\sqrt{n} + 2}{\sqrt{n+1}}$

f) $a_n = \frac{\sqrt{2n} + 5}{\sqrt{8n} - 1}$

g) $a_n = \frac{2^{n+1} - 3}{2^n + 1}$

h) $a_n = \frac{2n^2 + 1}{3n^2 + 4n}$

i) $a_n = \frac{1 - \sqrt{4n}}{3n}$

j) $a_n = \frac{n^2 + 2^n}{3\cdot 2^n}$

h) $a_n = \frac{\sin n}{n}$

a) $a_n = \frac{5}{n+4} = \frac{\overbrace{\frac{5}{n}}^{\to 0}}{1+\underbrace{\frac{4}{n}}_{\to 0}} \xrightarrow[n\to\infty]{} 0$

b) $a_n = \frac{2n}{3n-2} = \frac{2}{3-\underbrace{\frac{2}{n}}_{\to 0}} \xrightarrow[n\to\infty]{} \frac{2}{3}$

c) $a_n = \frac{6n+1}{2n-1} = \frac{6+\overbrace{\frac{1}{n}}^{\to 0}}{2-\underbrace{\frac{1}{n}}_{\to 0}} \xrightarrow[n\to\infty]{} 3$

d) $a_n = \frac{2\cdot 3^n+1}{3^{n-1}-2} = \frac{2+\overbrace{1/3^n}^{\to 0}}{1/3-\underbrace{\frac{2}{3^n}}_{\to 0}} \xrightarrow[n\to\infty]{} 6$

e) $a_n = \frac{\sqrt{n}+2}{\sqrt{n+1}} = \frac{1+\frac{2}{\sqrt{n}}}{\sqrt{\frac{n+1}{n}}} = \frac{1+\overbrace{2/\sqrt{n}}^{\to 0}}{\sqrt{1+\underbrace{\frac{1}{n}}_{\to 0}}} = 1$

f) $a_n = \frac{\sqrt{2n}+5}{\sqrt{8n}-1} = \frac{\sqrt{2}+\overbrace{\frac{5}{\sqrt{n}}}^{\to 0}}{\sqrt{8}-\underbrace{\frac{1}{\sqrt{n}}}_{\to 0}} \xrightarrow[n\to\infty]{} \frac{\sqrt{2}}{\sqrt{8}} = \frac{1}{2}$

g) $a_n = \frac{2^{n+1}-3}{2^n+1} = \frac{1-3/2^{n+1}}{1/2+1/2^{n+1}} \xrightarrow[n\to\infty]{} 2$

h) $a_n = \frac{2n^2+1}{3n^2+4n} = \frac{2+\frac{1}{n^2}}{3+\frac{4}{n}} \xrightarrow[n\to\infty]{} \frac{2}{3}$

i) $a_n = \frac{1-\sqrt{4n}}{3n} = \frac{\frac{1}{n}-\sqrt{\frac{4n}{n^2}}}{3} = \frac{\frac{1}{n}-\sqrt{\frac{4}{n}}}{3} \xrightarrow[n\to\infty]{} 0$

j) $a_n = \frac{n^2+2^n}{3\cdot 2^n} = \frac{\overbrace{\frac{n^2}{2^n}}^{0}+1}{3} \xrightarrow[n\to\infty]{} \frac{1}{3}$

a^n wächst schneller als $n^2, n^3, n^4, \ldots$
(für $a > 1$)

h) $a_n = \frac{\sin n}{n} \xrightarrow[n\to\infty]{} 0$ da $|\sin(n)| \le 1$